Economic Impact assessment of Entrepreneurship policies with the GMR-Europe Model

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List of Abbreviations

GMR	Geographic Macro Regional
DSGE	Dynamic Stochastic General Equilibrium
SCGE	Spatial Computable General Equilibrium
TFP	Total Factor Productivity
ENQ	Ego Network Quality
KP	Knowledge Potential
LS	Local Structure
LC	Local Connectivity
CC	Connected Components
REDI	Regional Entrepreneurship Development Index
PATSTCKR	Regional Petent Stock
PATSTCKN	National Patent Stock
HUMCAP	Human Capital
RD_TOTAL	Total Research and Development Expenditures
ENQFP	ENQ index calculated from Framework Program joint projects

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1 Executive summary

According to the recently introduces smart specialization policy concept regional specialization must grow out of the regions' own traditions instead of building on typically not replicable experiences of well-known successful regions. Instead of traditionally implemented sector-neutral innovation policy measures (e.g., human capital development or R&D support) and top-down policy tools targeting selected industries, the main instruments of smart specialization are a particular combination of these elements characterized by entrepreneurial discoveries supported by the government. As a result the entrepreneurial focus of this approach is a crucial point.

In this report we introduce the GMR-Europe policy impact model, which has been developed to facilitate impact assessment of smart specialization policies by specifically integrating variables describing the entrepreneurial ecosystem as well as the network embeddedness of European regions. The GMR-framework is rooted in different traditions of economics: in addition to modeling the spatial patterns of knowledge flows and the role of agglomeration in knowledge transfers it also accounts for interregional trade and migration in a general equilibrium context. The GMR models are structured around three model blocks. The Total Factor Productivity (TFP) block is able to capture the role of innovation-related factors such as R&D, human capital, entrepreneurship and knowledge networks in productivity growth at the regional level. A spatial computable general equilibrium (SCGE) block allows for the estimation of regional allocation and reallocation of resources as well as trade and migration as a result of given policy interventions. Finally, a macroeconomic (MACRO) model block generates the dynamics of key variables like employment, investment, capital stock. The complex interaction of these model blocks allows us to estimate the likely impacts of different policy interventions both at the regional and aggregate levels in several dimensions (GDP, productivity, employment, etc.). A novel feature of the present development of the GMR-Europe model is its capability to integrate policies targeting entrepreneurship. This is achieved by using the REDI index as a factor affecting regional productivity and through productivity also the economic development of the given region. Dynamic interactions through trade and factor mobility affect and feed back to the dynamics of other regions as well.

In addition to a detailed account of the model setup and estimation/calibration processes, this report also contains a brief simulation exercise illustrating the potential capabilities of the model in evaluating entrepreneurship-related policies. In the simulation we assume that a policy intervention improves the entrepreneurial climate/ecosystem in all regions of the model as measured by the REDI index. This intervention then affects regional productivities and through this regional economic output levels. We show that although the key driver of regional economic growth is productivity, there are differences as to what extent the same relative improvement affects regional productivities and also, the dynamic feedback mechanisms within the model generate diverse path for regional output levels in response to the shock implemented. The simulation in the report serves illustration purposes. Relying on the detailed structure of the REDI index it is possible to execute more fine-grained impact analyses with the model.

2 GMR-Europe: an overview

Developing entrepreneurship used to be a focal area in regional development policies: innovative activities behind, and employment created by, new firm formation are argued to be key drivers of regional economic prosperity. This focus was reinforced by the recently introduced, smart specialization policy concept. According to this approach specialization of a region must grow out of the regions' own traditions instead of building on typically not replicable experiences of well-known successful regions. Instead of traditionally implemented sector-neutral innovation policy measures (e.g., human capital development or R&D support) and top-down policy tools targeting selected industries, the main instruments of smart specialization are a particular combination of these elements characterized by entrepreneurial discoveries supported by the government. As a result a smart specialization policy combines the support of entrepreneurs to discover 'new domains of future opportunities' and the promotion of structural changes by prioritizing the ideas emerged from the region with non-neutrally designed policy instruments such as the promotion of human capital, R&D, entrepreneurship and knowledge network development. In sum, the entrepreneurial focus of this approach is a crucial point.

Although there are tools for estimating the likely impacts of traditional tools of development policy (like human capital development, support to research and development, development of infrastructure or investment supports), there is no established methodology for the evaluation of the likely effects of policies targeting entrepreneurship. Economic impact assessment targets the estimation of the likely impacts of given policies on economic variables like GDP, employment or inflation. Commonly applied instruments in economic impact evaluation are specifically designed economic models, there are however several challenges in using traditional models in the evaluation of smart specialization policies – which explains why economic impact assessment of smart specialisation programs has not been implemented in the cohesion policy framework. Two of the most important challenges in this respect are (i) integrating entrepreneurship and (ii) interregional network policies into an economic modelling framework are considered as the most prominent challenges.

In this report we give a detailed account of a modeling strategy which serves to overcome these challenges. We introduce the GMR-Europe policy impact model, which has been developed to facilitate impact assessment of innovation-related policies. Within the FIRES project we further developed the productivity block of the model in order to accommodate impact assessment of smart specialization policies by specifically integrating variables describing the entrepreneurial ecosystem as well as the network embeddedness of European regions. Using this setup we are able to analyze the potential economic effects of policies targeting entrepreneurship and/or networking on the regional, national or EU level.

This part of the report provides a general overview of the GMR-Europe model. The details of the three model blocks are exposed in part 3, while part 4 provides an illustrative simulation describing the potential use of the model in evaluating regional entrepreneurship support policies.

2.1 General features of GMR models

The geographic macro and regional modeling (GMR) framework has been established and continuously improved to better support development policy decisions by ex-ante and ex-post scenario analyses. Policy instruments including R&D subsidies, human capital development, entrepreneurship policies or instruments promoting more intensive public-private collaborations in innovation are in the focus of the GMR-approach.

Models frequently applied in development policy analysis are neither geographic nor regional. They either follow the tradition of macroeconometric modeling (like the HERMIN model - ESRI 2002), the tradition of macro CGE modeling (like the ECOMOD model – Bayar 2007) or the most recently developed DSGE approach (QUEST III - Ratto, Roeger and Veld 2009). They also bear the common attribute of national level spatial aggregation. The novel feature of the GMR-approach is that it incorporates geographic effects (e.g., agglomeration, interregional trade, migration) while both macro and regional impacts of policies are simulated. Why does geography get such an important focus in the system? Why the system is called “regional” and “macro” at the same time?

Geography plays a critical role in development policy effectiveness for at least four major reasons. First, interventions happen at a certain point in space and the impacts might spill over to proximate locations to a considerable extent. Second, the initial impacts could significantly be amplified or reduced by short run (static) agglomeration effects. Third, cumulative long run processes resulting from labor and capital migration may further amplify or reduce the initial impacts in the region resulting in a change of the spatial structure of the economy (dynamic agglomeration effects). Forth, as a consequence of the above effects different spatial patterns of interventions might result in significantly different growth and convergence/divergence patterns.

“Regions” are spatial reference points in the GMR-approach. They are sub-national spatial units ideally at the level of geographic aggregation, which is appropriate to capture proximate relations in innovation. Besides intraregional interactions the model captures interregional connections such as knowledge flows exceeding the regional border (scientific networking or spatially mediated spillovers), interregional trade connections and migration of production factors.

Important regional dimensions that may crucially determine the growth effects of development policies include the following aspects.

- Regional development programs are built on important *local specificities* (industrial structure, research strengths of the region, size and specialization of human capital etc.).
- Models have to capture the effects of policies on *local sources of economic growth* such as technological progress, investment and employment.
- The models also need to be able to follow those cumulative *agglomeration impacts* such as intensifying localized knowledge spillovers and their feedback mechanisms that may arise as a consequence of policies.
- There are certain additional impacts on the regional economy instrumented by *Keynesian demand side* effects or *Leontief-type intersectoral* linkages.
- Most of the infrastructural programs target better physical *accessibility*. Impacts of these policies on regions that are (directly or indirectly) affected also have to be reflected.

- There are different mechanisms through which policies implemented in certain regions affect other territories such as *interregional knowledge spillovers and trade linkages* and as such these effects also need to be incorporated in model structures.

The “macro” level is also important when the impact of development policies is modeled: fiscal and monetary policy, national regulations or various international effects are all potentially relevant factors in this respect. As a result the model system simulates the effects of policy interventions both at the regional and the macroeconomic levels. With such an approach different scenarios can be compared on the basis of their impacts on (macro and regional) growth and interregional convergence.

The GMR-framework is rooted in different traditions of economics (Varga 2006). While modeling the spatial patterns of knowledge flows and the role of agglomeration in knowledge transfers it incorporates insights and methodologies developed in the geography of innovation field (e.g., Anselin, Varga and Acs 1997, Varga 2000). Interregional trade and migration linkages and dynamic agglomeration effects are modeled with an empirical general equilibrium model in the tradition of the new economic geography (e.g., Krugman 1991, Fujita, Krugman and Venables 1999). Specific macroeconomic theories are followed while modeling macro level impacts.

The first realization of the GMR approach was the EcoRET model built for the Hungarian government for ex-ante and ex-post evaluation of the Cohesion policy (Schalk and Varga 2004). This was followed by the GMR-Hungary model, which is currently used by the Hungarian government for Cohesion policy impact analyses (Varga 2007). GMR-Europe was built in the IAREG FP7 project (Varga, Járosi, Sebestyén 2011, Varga 2017) and further developed in the GRINCOH FP7 project (Varga, Járosi, Sebestyén, Szerb 2015). The most recent version of GMR-models is GMR-Turkey (Varga, Járosi, Sebestyén, Baypinar 2013, Varga and Baypinar 2016).

GMR models reflect the challenges of incorporating regional, geographic and macroeconomic dimensions in development policy impact modeling by structuring the system around the mutual interactions of three sub-models such as the Total Factor Productivity (TFP), Spatial Computable General Equilibrium (SCGE) and macroeconomic (MACRO) model blocks. Following this approach the macroeconomic model of GMR-Europe calculates policy impacts at the national level while the 181 NUTS 2-level regional models provide results at the regional level.

Some policy interventions can be modeled in the macroeconomic block (such as changes in international trade, in tax regulations or in income subsidies) via policy shocks affecting specific macroeconomic equations. However, many other policy instruments may apply on the regional level, stimulating the regional base of economic growth such as investment support, infrastructure building, human capital development, R&D subsidies, promotion of (intra- and interregional) knowledge flows. These interventions are modelled in the regional model blocks and also interact with the macroeconomic part. In the following sub-section we focus on mechanisms of these latter policies.

2.2 Regional impact mechanisms of the main policy variables

2.2.1 R&D support, interregional knowledge networks, human capital and entrepreneurship

Figure 1 after some pages shows the way how impacts of policies targeting R&D support, interregional knowledge networks, human capital and entrepreneurship are modeled in the GMR model (section 3.1 will give a detailed description on this part of the model). The regional level of economically useful new knowledge is measured by patents in the model. R&D support and interregional knowledge networks affect the economy via their impact on regional patenting. Increasing patenting activity may in turn affect the regions' general technological levels which then contribute to higher productivity, captured by total factor productivity (TFP) in the model. Productivity, on the other hand is affected by the regional level of human capital and the quality of entrepreneurial environment.

The GMR-Europe v2 model contains a novel element in its productivity-development block, namely entrepreneurship. We assume that entrepreneurship positively affects regional productive possibilities through enhancing the effectiveness of human capital in the region. We implement entrepreneurship in the model through the REDI index (Regional Entrepreneurship and Development Index). Through this element of the model it becomes possible to simulate the regional and macroeconomic impacts of policies which aim at supporting entrepreneurship at the regional level. The details in this respect are discussed in section 3.1.3.

The impacts of the promotion of R&D, networking, human capital and entrepreneurship on economic variables (prices of quantities of inputs and outputs, etc.) are calculated in the SCGE block. Economic impacts of increased productivity are modeled in the SCGE block in the following steps.

2.2.1.1 Short run effects

The impact in the short run results from the interplay between the substitution and output effects. Assuming that the level of production does not change the same amount of output can be produced by less input that is the demand for capital (K) and labor (L) decrease as a result of the interventions. However increased TFP makes it also possible to decrease prices to keep firms more competitive, which positively affects demand. This latter effect is called the output effect. The interaction of output and substitution effects might result in the increase of the demand for factor inputs (K and L) but also the impact can be just the opposite. What will actually happen is an empirical question. In case output effect exceeds substitution effect wages will increase in the short run, which together with the relative decrease in prices will result in increasing consumption and higher utility levels.

2.2.1.2 Long run effects

Increased utility levels result in in-migration of labor and capital into the region, which will be the source of further cumulative effects working via centripetal and centrifugal forces. Labor migration increases employment concentration, which is a proxy for positive agglomeration effects in the model. According to findings in the literature localized knowledge spillovers intensify with the concentration of economic activity in the region (e.g., Varga 2000). A higher level of employment thus increase TFP (as shown also in Figure 1), which further reinforces in-migration of production factors following the mechanisms described above. However increasing population also affect the average size of flats negatively which work as a centrifugal force in the model. The balance between centrifugal and centripetal forces will determine the long term cumulative effect of policies at the regional, interregional and macroeconomic levels.

2.2.2 Infrastructure investments

Infrastructure investments increase the level of public capital in the region. It is modeled via a Cobb-Douglas production function where the inputs are labor, private and public capitals. Thus infrastructure investments are modeled as externalities, which eventually affect regional TFP level. Public investments are also modeled in the macro model via the increase of public capital.

2.2.3 Private investment support

One of the policies suggested is the support of investment by small and medium sized enterprises. The mechanism of this policy instrument affects the model via the increase in private capital, which has further impacts on several other variables both in the region where the intervention occurs and in other regions connected by trade or migration linkages. Private investment support is also modeled in the macro model via the increase of private capital.

2.3 Macroeconomic impacts

The effects of policies are communicated to the macro model by changes in TFP (aggregated from the regional level) and changes in fiscal variables (such as the demand and supply impacts of investment support and physical infrastructure construction). Changing TFP results in an increase of GDP growth rate which, will increase factor demand resulting from their higher marginal productivities. As a result the level of GDP will be higher than what would be observed in its long run equilibrium path. Infrastructure investments and private investment support induce both demand and supply side effects. The demand side (e.g., increased government expenditures) effect on GDP is temporary while the supply side effects (via increased public and private capitals) stabilize in the long run.

2.4 Impact mechanisms in the GMR model

The mutually connected three model-block system is depicted in Figure 1 below. Without interventions TFP growth rate follows the national growth rate in each region. The impacts of interventions run through the system according to the following steps.

1. Resulting from R&D-related interventions as well as human capital and physical infrastructure investments (which increase public capital and eventually impact the level of TPF as well) regional Total Factor Productivity increases.

2. Changing TFP induces changes in quantities and prices of output and production factors in the short run while in the long run (following the mechanisms described above) the impact on in-migration of production factors imply further changes in TFP not only in the region where the interventions happen but also in regions which are connected by trade and factor migration linkages.

3. Increased private investments expand regional private capital which affects further changes in regional variables (output, prices, wages, prices, TFP, etc.) in the SCGE model block. The impact of private investment support affects the macro model as well via increased private capital.

4. For each year changes in TFP are aggregated to the national level then this increases TFP in the macro model as time specific shocks. The macroeconomic model calculates the changes in all affected variables at the national level.

5. Changes in employment and investment calculated in the MACRO block are distributed over the regions following the spatial pattern of TFP impacts.

6. The SCGE model runs again with the new employment and capital values to calculate short run and long run equilibrium values of the affected variables.

7. The process described in steps 5 and 6 run until aggregate values of regional variables calculated in the SCGE model get very close to their corresponding values calculated in the MACRO model.

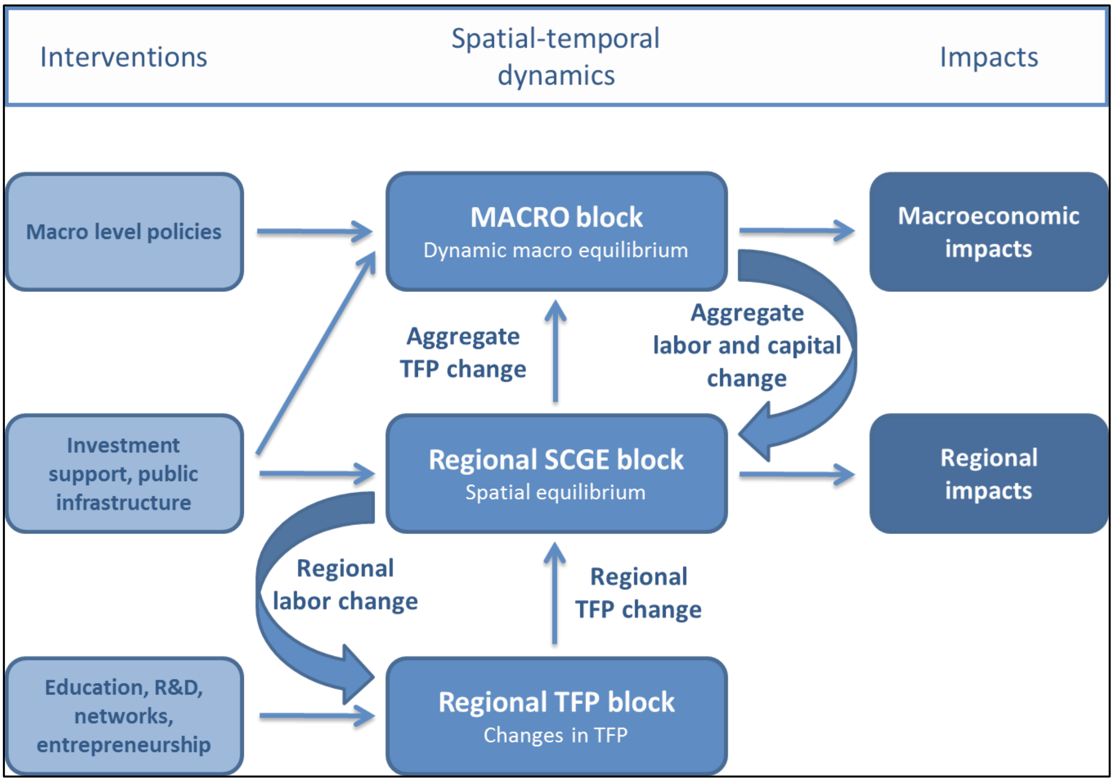

Figure 1 – Regional and macroeconomic impacts of the main policy variables in the GMR-Europe model

3 GMR model blocks

As it was shown in Figure 1, the GMR model consists of three main building blocks, the general connections of which were exposed in the previous part. In this part, we provide a detailed description of each model block in turn. We give the equations and the data for each. Section 3.1 is devoted to the TFP block, section 3.2 deals with the SCGE block and section 3.3 discusses the MACRO block.

3.1 The TFP model block

TFP is one of the most crucial variables in GMR-Europe. It represents the main point through which different aspects of innovation and innovation policy interventions in particular interact with other parts of the model. The TFP block serves as the point in the GMR system where “soft” and “hard” factors behind innovation are modelled. Then, in line with the traditions in economic modelling, all these factors are implemented in the MACRO and SCGE blocks through one technology variable, generally referred to as total factor productivity.

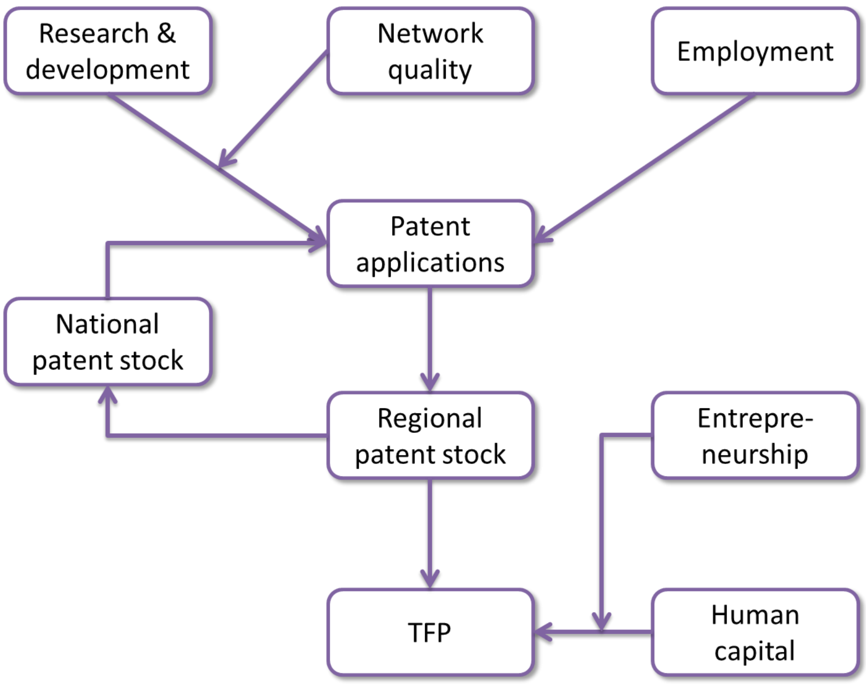

Figure 2 – The schematic structure of the TFP block

Figure 2 illustrates the setup of the TFP block in GMR-Europe. TFP is the final variable down in the middle which then transfers impacts generated in the TFP block over to the other parts of the model, namely the SCGE block and the MACRO block. However, the main role of the TFP block is to provide a sophisticated background for determining TFP and implement innovation-oriented policy interventions. The TFP block is based on the knowledge production function approach, where new knowledge, represented by patent applications in our model setup, is produced using knowledge production factors, namely R&D efforts and labor (employment), as well as already existing knowledge which is represented by national patent stock (knowledge creation and TFP is directly modelled at the regional level). In addition to this standard approach, we also include the role of knowledge available through interregional networks through a special index Ego Network Quality (see details later) which is assumed to affect the productivity of R&D in knowledge creation (better network positions lead to higher knowledge output for the same amount of inputs). New knowledge, i.e. patent applications at the regional level then feed back into knowledge creation in a dynamic way by building up national patent stock.

TFP is primarily linked to the regional knowledge levels in the model (described just before), but two additional forces are added to the determination of regional TFP. First, the level of human capital in the region affects TFP and second, a focal element of this setup of the GMR model, we added the entrepreneurial environment in the model which is also assumed to have a positive influence on TFP, via enhancing the contribution of human capital to TFP. The argument behind this setting is that a better entrepreneurial climate in a region mobilises regional human capital to get increasingly engaged in entrepreneurial activities, which eventually leads to increasing total factor productivity.

The argument behind this setting is that a better entrepreneurial climate in a region helps better exploit the possibilities lying in human capital (e.g. providing opportunities for creative work through enhancing job opportunities in the region for workforce with higher educational levels where they can effectively use their knowledge).

For the estimation of the equations of the TFP block, we need data on all variables listed in Figure 2. Some of these are straightforward to use and are available from standard sources (see section 3.1.4.3 for the details on the data). These are R&D expenditures, employment, patent applications, patent stocks and human capital. We have three variables, however, which need further elaboration to be used in the equations. These are the quality of interregional knowledge networks, the entrepreneurial environment in the regions and the TFP itself. In this section we provide a brief discussion of each.

3.1.1 Estimating TFP for European regions

Total factor productivity is the overall productivity of production factors and can be estimated or recalculated from an appropriately specified production function. In estimating the TFP the production function is assumed to be of the Cobb-Douglas type with constant returns to scale for capital and labor:

$$Y_{it} = TFP_{it} \cdot K_{it}^{\alpha} \cdot L_{it}^{1-\alpha} \qquad \text{(T1)}$$

where Y_{it} is gross value added, L_{it} is labor, K_{it} is the capital stock in region i at time t. This function is used to calculate TFP values for all regions and all years. For this, we need data on employment, output and the capital stock as well as the elasticit Regional employment and GDP data are used directly from Eurostat and capital stocks were estimated using the Perpetual Inventory Method (PIM) as in many cases in the literature. This method uses the average growth rate of investments and depreciation rate of the capital. To do so, long time series of regional investments are required. That is why this method is often used in calculating capital stocks at the national level. Here, we use the method to produce time series for capital stocks at the regional level.

First, we obtained investment data for the EU NUTS 2 regions between 2000 and 2012. Unfortunately there were several holes in this dataset thus we gathered further data from national statistical databases too. After that we transformed the current price investment to constant price PPP data. Then we estimated the average growth rate between 2000 and 2005 to and set the depreciation rate to 10% in order to generate the baseline capital stock for 2005 for each NUTS 2 regions. After that we adjusted this regional capital stock data to represent the national net capital stock from AMECO database (which is also converted to PPP data). Then using the depreciation rate and regional investment we calculated regional capital stocks for the next years and we always paid attention to adjust them according to the converted AMECO data. In the end we obtained the capital stock data for all regions for 2012.

Second, we used the regional employment, GDP and the estimated capital stock to estimate the regional total factor productivity. For this, we used national production elasticities calculated as the share of the compensation of employees in national value added from national input-output tables. At this point we had all elements in the production function necessary to calculate the TFP for each region and time period according to the following formula:

$$TFP_{it} = Y_{it} / \left(K_{it}^{\alpha} \cdot L_{it}^{1-\alpha} \right) \quad \text{(T2)}$$

3.1.2 Measuring extra-regional knowledge accessed via research networks: The Ego Network Quality (ENQ) index

In the following empirical analyses we employ the Ego network Quality index developed and introduced by Sebestyén and Varga (2013a, 2013b), in order to capture the amount of knowledge available by a region through its interregional knowledge connections. The concept of ENQ builds on three intuitions directly influenced by the theory of innovation. First, that the level of knowledge in an agent's network is in a positive relationship with the agents' productivity in generating new knowledge. Second, that the structure of connections in the agents' network can serve as an additional source of value (see e.g. Coleman 1986; Burt 1992). Third, that partners in the ego network contribute to diversity through building connections to different further groups not linked directly to the agent.

The ENQ index is structured around two dimensions, which are then augmented with a related third aspect. The two dimensions are: (i) Knowledge Potential, which measures knowledge accumulated in the direct neighbourhood and it is related to the number of partners and the knowledge of individual partners, and (ii) Local Structure, what is associated with the structure of links among partners. The third aspect is Global Embeddedness (GE) and captures the quality of distant parts of the network (beyond immediate partners). However, this aspect is implemented by applying the concepts of KP and LC for consecutive neighbourhoods of indirect partners in the network.[1] Here we give a brief summary of the ENQ index with the most important aspects. The reader is directed to Sebestyén and Varga (2013a, 2013b) for more detailed discussion.

The network under consideration is represented by the adjacency matrix $\mathbf{A} = [a_{ij}]$, where the general element a_{ij} describes the connection between nodes i and j. The adjacency matrix defines the matrix of geodesic distances (lengths of shortest paths) between all pairs of nodes, which we

[1] By 'neighbourhood at distance d' we mean the nodes exactly at distance d from a specific node.

denote by $\mathbf{R} = [r_{ij}]$. In order to account for knowledge levels, we use $\mathbf{k} = [k_i]$ as the vector of knowledge at each specific node of the network.

We formalize the conceptual model of ENQ presented above in the following way:

$$ENQ^i = \sum_{d=1}^{M-1} W_d LS_d^i \, KP_d^i = LS_1^i KP_1^i + GE^i \quad \text{(T3)}$$

where superscript i refers to the node for which ENQ is calculated and subscript d stands for distances measured in the network (geodesic distance). M is the size of the network, W_d is a weighting factor used for discounting values at different d distances from node i,[2] whereas KP_d^i and LS_d^i are the respective Knowledge Potential and Local Structure values evaluated for the neighbourhood at distance d from node i. The proposed formula can be interpreted as calculating the Knowledge Potentials for neighbourhoods at different distances from node i, weighted by the Local Structure value of the same neighbourhood. Then, these results for the different neighbourhoods are weighted by a distance-decay factor and summed over distances. The second equation in the above formula shows (using $W_1 = 1$ by definition) how the ENQ index can be divided into the three dimensions mentioned above: the Knowledge Potential and the Local Structure of the direct neighbourhood and Global Embeddedness which sums these aspects beyond the direct neighbourhood. In what follows, the two basic concepts, Knowledge Potential and Local Structure are introduced in more detail.

3.1.2.1 Knowledge Potential

Using the notation presented before, the concept of KP can be formulated in the following way:

$$KP_d^i = \sum_{j:r_{ij}=d} k_j \quad \text{(T4)}$$

The Knowledge Potential, as perceived by node i, can thus be calculated for the neighbourhoods at different d distances from node i, and for all these distances it is the sum of knowledge possessed by nodes at these distances.

3.1.2.2 Local Structure

The concept of Local Structure refers to the structure of connections in different neighbourhoods of a node. What one means by structure, though, is a matter of question here. In this paper we introduce two specific ways to fill LS with content, namely Local Connectivity and Connected Components. The two alternative specifications are linked to the concepts of cohesion and structural holes familiar from the theory of social capital. Cohesion, as defined by Coleman (1986) emphasizes the role of cohesion, while the notion of structural holes (Burt 1992) puts weight on gatekeepers or information brokers connecting different groups in the network.

Local Connectivity

Local Connectivity (LC), referring to the cohesion concept, is associated with the strength of ties and the intensity of interactions among partners. It is the sum of the tie weights present in a given neighbourhood, normalized by the size of this neighbourhood:

$$LC_d^i = \frac{1}{N_d^i}\left(\sum_{j:r_{ij}=d-1} \sum_{l:r_{il}=d} a_{jl} + \frac{\sum_{j:r_{ij}=d} \sum_{l:r_{il}=d} a_{jl}}{2}\right) \quad \text{(T5)}$$

where N_d^i is the number of nodes laying exactly at distance d from node i. The first term in the parenthesis counts the (possibly weighted) ties between nodes at distance $d-1$ and d.[3] This reflects the intensity at which two adjacent neighbourhoods are linked together. The second term counts the

[2] In this paper we apply exponential weighting, where $W(d) = e^{1-d}$. Some analysis with respect to different formulations can be found in Sebestyén and Varga (2013b).

[3] Distances are always measured from node i.

(possibly weighted) number of ties among nodes at distance d.[4] As a result, Local Connectivity can be defined as intensity with which the (possibly indirect) neighbours at distance d are linked together and linked to other neighbourhoods. Using the LC approach, the ENQ index is formulated as follows:

$$ENQ^i = \sum_d W_d Q_d^i = \sum_d W_d KP_d^i LC_d^i \qquad \text{(T6)}$$

Connected Components

Connected Components (CC) integrates the concept of structural holes into the ENQ index through LS. Here we propose a simple approach to capture the basic intuition behind the concept: we introduce CC_d^i which counts the number of connected components (unconnected groups of nodes) in different neighbourhoods.[5] Using the CC approach, the ENQ index is formulated as follows:

$$ENQ^i = \sum_d W_d Q_d^i = \sum_d W_d KP_d^i CC_d^i \qquad \text{(T7)}$$

A mixed version

Although both intuitive, Local Connectivity and Connected Components take a very strict view and measurement of the phenomena they intend to capture. However, by combining the two approaches, ENQ can reflect a more refined picture about the structure of local neighbourhoods. Let's redefine ENQ with the product of Local Connectivity and Connected Components as the weighting factor of Knowledge Potentials (the Local Structure component, defined before):

$$ENQ^i = \sum_d W_d Q_d^i = \sum_d W_d KP_d^i CC_d^i LC_d^i \qquad \text{(T8)}$$

This formulation refines the two extreme cases by providing a natural way to combine the two effects as the multiplication of Connected Components and Local Connectivity attach higher weights to structures which lay in between neighborhoods with extreme structural holes and extreme connectivity.

3.1.3 Measuring regional entrepreneurship: The REDI index[6]

In the estimation of the TFP block equations, an important novel addition to the GMR approach is the inclusion of entrepreneurial environment at the regional level. In order to include this aspect to the model, we apply the Regional Entrepreneurship and Development Index as a measure for entrepreneurship at the regional level. In what follows, we provide a brief discussion of the index.

3.1.3.1 The structure of REDI

The Regional Entrepreneurship and Development Index (REDI) has been constructed for capturing the contextual features of entrepreneurship across EU regions. The REDI method builds on the National Systems of Entrepreneurship Theory and provides a way to profile Regional Systems of Entrepreneurship. Important aspects of the REDI method including the Penalty for Bottleneck (PFB) analysis, which helps identifying constraining factors in the Regional Systems of Entrepreneurship. The novelty of this method that it portrays the entrepreneurial disparities amongst EU regions and

[4] Division by two is required because matrix $\mathbf{A}$ is symmetric, and thus we can avoid duplications in the counting.
[5] The number of connected components in a neighbourhood is given by the multiplicity of the zero eigenvalues of the Laplacian matrix of the subgraph spanned by the nodes at a specific distance from the node in question (see e.g. Godsil and Royle 2001).
[6] This section draws on Szerb et al. (2017), various sections.

provides country and regional level, tailor-made public policy suggestions to improve the level of entrepreneurship and optimize resource allocation over the different pillars of entrepreneurship.

A six level index-building methodology is followed while creating the REDI index: (1) sub-indicators (2) indicators (3) variables, (4) pillars, (5) sub-indices, and finally (6) the REDI super-index. The three sub-indices of attitudes (ATT), abilities (AB), and aspiration (ASP) constitute the entrepreneurship super-index, which is called REDI. All three sub-indices contain four or five pillars, which can be interpreted as quasi-independent building blocks of this entrepreneurship index. Each of the 14 pillars is the result of the multiplication of an individual variable and an associated institutional variable. In this case, institutional variables can be viewed as particular (regional-level) weights of the individual variables. Figure 3 provides a detailed picture of the sub-indices.

	Sub-indexes	Pillars	Variables (ind./inst.)
REGIONAL ENTREPRENEURSHIP INDEX	ATTITUDES SUB-INDEX	OPPORTUNITY PERCEPTION	*OPPORTUNITY RECOGNITION*
			MARKET AGGLOMERATION
		STARTUP SKILLS	*SKILL PERCEPTION*
			QUALITY OF EDUCATION
		RISK ACCEPTANCE	*RISK PERCEPTION*
			BUSINESS RISK
		NETWORKING	*KNOW ENTREPRENEUR*
			SOCIAL CAPITAL
		CULTURAL SUPPORT	*CARRIER STATUS*
			OPEN SOCIETY
	ABILITIES SUB-INDEX	OPPORTUNITY STARTUP	*OPPORTUNITY MOTIVATION*
			BUSINESS ENVIRONMENT
		TECHNOLOGY ADOPTION	*TECHNOLOGY LEVEL*
			ABSORTIVE CAPACITY
		HUMAN CAPITAL	*EDUCATIONAL LEVEL*
			EDUCATION AND TRAINING
		COMPETITION	*COMPETITORS*
			BUSINESS STRATEGY
	ASPIRATION SUB-INDEX	PRODUCT INNOVATION	*NEW PRODUCT*
			TECHNOLOGY TRANSFER
		PROCESS INNOVATION	*NEW TECHNOLOGY*
			TECHNOLOGY DEVELOPMENT
		HIGH GROWTH	*GAZELLE*
			CLUSTERING
		GLOBALIZATION	*EXPORT*
			CONNECTIVITY
		FINANCING	*INFORMAL INVESTMENT*
			FINANCIAL INSTITUTIONS

Figure 3 – The structure of the Regional Entrepreneurship Development Index

3.1.3.2 The creation of the Regional Entrepreneurship and Development Index

All pillars from the variables are calculated using the interaction variable method; that is, by multiplying the individual variable with the proper institutional variable:

$$z_{i,j} = IND_{i,j} * INS_{i,j} \quad \text{(T9)}$$

for all $j = 1, \dots, k$, the number of individual and institutional variables. $IND_{i,j}$ is the original score value for region i and variable j individual variable, $INS_{i,j}$ is the original score value for region i and variable j institutional variable, $z_{i,j}$ is the original pillar value for region i and pillar j.

All index building is based on a benchmarking principle. The selection of the proper benchmarking considerably influences the index points and also the rank of the regions. However, the existence of outliers could lead to setting up inappropriate benchmarks. Hence, it is needed to handle extreme value outliers. Capping is frequently used to handle outliers, where the question relates to the value of the cap. In our case we selected the 95-percentile score adjustment, meaning that any observed value higher than the 95 percentile is lowered to the 95 percentile. It also means that at least five percent of different regions reach the maximum value in all of the 14 pillars. Like other composite index components, the pillars are recorded in different magnitudes. In order to be in exactly the same range, the normalization of the pillars is necessary. After handling the outliers the pillar values are normalized, where the distance normalization technique was used that preserves the distance (relative differences) amongst the regions:

$$x_{i,j} = \frac{z_{i,j}}{\max z_{i,j}} \qquad \text{(T10)}$$

for all $j = 1, .. m. m = 14$ is the number of pillars, $x_{i,j}$ is the normalized score value for region i and pillar j, $z_{i,j}$ is the original pillar value for region i and pillar j, $\max_i z_{i,j}$ is the maximum value for pillar j.

Applying the distance methodology the pillar values are all in the range [0,1], but the lowest pillar value is not necessary equal to 0. In this case all regions' efforts are evaluated in relation to the benchmarking region but the worst region is not set to zero per se.

The different averages of the normalized values of the 14 pillars imply that reaching the same performance requires different effort and consequently resources. Higher average values - e.g. Opportunity startup – could mean that it is easier to reach better scores as compared to lower average value – e.g. Financing. Since the aim is to apply REDI for public policy purposes, the additional resources for the same marginal improvement of the pillar values should be the same for all of the 14 pillars, on the average. So improving by 0.1 unit Opportunity startup should require the same additional resource as compared to all the other 13 pillars. As a consequence, we need a transformation to equate the average values of the 14 pillars.

Practically we have calculated the average values of the 14 pillars after the capping adjustment and the normalization and made the following average adjustment. Let x_i be the normalized score for region i for a particular pillar j.

The arithmetic average of pillar j for region n regions is:

$$\bar{x}_j = \frac{\sum_{i=1}^{n} x_{i,j}}{n} \qquad for\ all\ j \qquad \text{(T11)}$$

We want to transform the $x_{i,j}$ values such that the potential values lay in the [0,1] range.

$$y_{i,j} = x_{i,j}^{k} \qquad \text{(T12)}$$

where k is the "strength of adjustment", the k^{th} moment of X_j is exactly the needed average, $\bar{y}_j$. We have to find the root of the following equation for k:

$$\sum_{i=1}^{n} x_{i,j}^{k} - n\bar{y}_j = 0 \quad \text{(T13)}$$

It is easy to see based on previous conditions and derivatives that the function is decreasing and convex which means it can be quickly solved using the well-known Newton – Raphson method with an initial guess of 0. After obtaining k, the computations are straightforward. Note that if

$$\bar{x}_j < \bar{y}_j \quad k < 1$$

$$\bar{x}_j = \bar{y}_j \quad k = 1 \quad \text{(T14)}$$

$$\bar{x}_j > \bar{y}_j \quad k > 1$$

that is k be thought of as the strength (and direction) of adjustment.

We have defined entrepreneurship as the interaction of entrepreneurial attitudes, abilities, and aspirations and developed the **Penalty for Bottleneck (PFB)** methodology for measuring and quantifying these interactions (Acs et al., 2013a; Rappai and Szerb 2011). Bottleneck is defined as the worst performing weakest link, or binding constraint in the system. With respect to entrepreneurship, by bottleneck we mean a shortage or the lowest level of a particular entrepreneurial indicator as compared to other indicators of the sub-index. This notion of bottleneck is important for policy purposes. Our model suggests that attitudes, ability and aspiration interact, and if they are out of balance, entrepreneurship is inhibited.

The sub-indices are composed of four or five components, defined as indicators that should be adjusted in a way that takes this notion of balance into account. After normalizing the scores of all the indicators, the value of each indicator of a sub-index in a region is penalized by linking it to the score of the indicator with the weakest performance in that region. This simulates the notion of a bottleneck, and if the weakest indicator were improved, the particular sub-index and ultimately the whole REDI would show a significant improvement. To the contrary, improving a relatively high pillar value will presumably enhance only the value of the pillar itself, and in this case a much smaller increase of the whole REDI index can be anticipated. Moreover, the penalty should be higher if differences are higher. Looking from either the configuration or the weakest link perspective it implies that stable and efficient sub-index configurations are those that are balanced (have about the same level) in all indicators. Mathematically, we model the penalty for bottlenecks by modifying Casado-Tarabusi and Palazzi (2004) original function for our purposes. The penalty function is defined as:

$$h_{(i),j} = min\, y_{(i),j} + \left[1 - e^{-(y_{(i)j} - min y_{(i),j})}\right] \quad \text{(T15)}$$

where $h_{i,j}$ is the modified, post-penalty value of pillar j in region i, $y_{i,j}$ is the normalized value of index component j in region i, y_{min} is the lowest value of $y_{i,j}$ for region i. $i = 1, 2, \ldots\ldots n$ is the number of regions, $j = 1, 2, \ldots m$ is the number of pillars.

Definitely, the advantage of this method that it is an analytical method, therefore it is not sensitive to the size of the sample. There are two potential drawbacks of the PFB method. One is the arbitrary selection of the magnitude of the penalty. The other problem is that we cannot exclude fully the potential that a particularly good feature can have a positive effect on the weaker performing features. While this could also happen, most of the entrepreneurship policy experts hold that policy should focus on improving the weakest link in the system. On the other hand, both theories emphasize the importance of balanced performance and characteristics. Altogether, we claim that the PFB methodology is theoretically better than the arithmetic average calculation. However, the PFB adjusted REDI is not necessary an optimal solution since the magnitude of the penalty is unknown. The most important message for economic development policy is that improvement can only be achieved by abolishing the weakest link of the system, which has a constraining effect on other pillars.

Due to the average pillar adjustment the marginal rate of substitution becomes the same for all indicators. However, the real substitution rate of the pillar values of a particular region depends on the weakest pillar's relative ratio compared to other pillars. Most importantly, the penalty function should reflect to the magnitude of the penalty, lower difference implies lower penalty while higher unbalance implies higher penalty. The penalty function also reflects to the compensation of the loss of one pillar for a gain in another pillar.

The value of a sub-index for any region was then calculated as the arithmetic average of its PFB-adjusted indicators for that sub-index multiplied by 100 to get a 100 point scale:

$$ATT_i = 100 \sum_{j=1}^{5} h_j \tag{T16}$$

$$ABT_i = 100 \sum_{j=6}^{10} h_j \tag{T17}$$

$$ASP_i = 100 \sum_{j=11}^{14} h_j \tag{T18}$$

where $h_{i,j}$ is the modified, post-penalty value of pillar j in region i, $i = 1, 2, \ldots \ldots n$ is the number of regions, $j = 1, 2, \ldots m$ is the number of pillars.

The REDI super-index is simply the arithmetic average of the three sub-indices:

$$REDI_i = \frac{1}{3}(ATT_i + ABT_i + ASP_i) \tag{T19}$$

where $i = 1, 2, \ldots \ldots n$ is the number of regions.

3.1.4 Equations in the TFP block and their estimation

The TFP block of the model, as indicated by Figure 2, consists of two equations: one is a knowledge production function which links new knowledge, measured by regional patent applications, to knowledge inputs. The other one is the TFP equation which links, among others, regional knowledge to TFP. In what follows, we provide the two equations and their estimation in turn.

3.1.4.1 The TFP equation

The two equations of the TFP block are estimated separately. In line with Figure 2 we assume that the level of TFP depends on three central factors. Knowledge accumulated over the past years, human capital and entrepreneurship, the latter contributing to the effectiveness of human capital. In the estimated equation entrepreneurship therefore enters in interaction with human capital. The following equation is estimated:

$$\log(TFP_{t,r}) = \alpha + \beta_1 \log(PATSCKR_{t-1,r}) + \beta_2 \log(HUMCAP_{t-1,r}) + \beta_3 \log(HUMCAP_{t-1,r}) \log(REDI_{t-1,r}) + \varepsilon_{t,r} \quad \text{(T20)}$$

where t refers to time periods and r refers to region indices. Accumulated knowledge is measured by the cumulative number of patents (PATSCKR) while the level of human capital at regional level is proxied by the population (between age 25-64) with tertiary education attainment (HUMCAP). Entrepreneurship is measured by the REDI index as discussed in section 3.1.3 (REDI).

3.1.4.2 The patent equation (knowledge production function)

According to Figure 2, the patent application intensity of a region is explained by the national patent stock, research and development efforts, employment in the region and knowledge network embeddedness, the latter contributing to the effectiveness of research and development efforts. As in the TFP equation, this latter effect is modelled by interacting research and development with network quality in the estimated model. The following equation is estimated:

$$\log(PAT_{t,r}) = \alpha + \beta_1 \log(PATSCKN_{t-1,N}) + \beta_2 \log(EMP_{t-1,r}) + \beta_3 \log(RD\_TOTAL_{t-1,r}) + \beta_4 \log(RD\_TOTAL_{t-1,r}) \log(ENQFP_{t-1,r}) + \varepsilon_t \quad \text{(T21)}$$

Patents (knowledge) on the left hand side is measured by EPO patent applications (PAT), national patent stock is the cumulated number of patents at the country level (PATSCKN), research and development efforts are proxied with R&D expenditures (RD_TOTAL), employment is captured by the total level of employment in the region (EMP) and network quality is measured with the ENQ index calculated over the network of Framework Program partnerships between regions (ENQFP).

3.1.4.3 The TFP block database

Table 1 contains the data sources for estimating the production function (TFP calculation)

Table 1 – Data sources of the production function (TFP calculation)

Variable name	Description	Source
$GDP_{i,t}$	Gross domestic product (GDP) at current market prices by NUTS 2 regions (nama_r_e2gdp)	Eurostat
$L_{i,t}$	Employment by economic activity and NUTS 2 regions	Eurostat
$K_{i,t}$	Regional net capital stock (private)	Authors' own calculations as described in section 3.1.1

For the production function we first estimated the regional GDP in PPP based on 2000 prices. In order to do this we first obtained the current and PPP price national GDP from Eurostat between

1995 and 2009. Then using these sources we calculated a ratio (PPP / current price) that shows us how to convert a given year's current price GDP into a PPP GDP data. Then we used this national ratio to convert all regional current price GDP into constant price PPP GD

Then for employment we used unadjusted Eurostat data. In some cases we had to refer to the database of national statistical offices since there were no available data at Eurostat.

The calculation of private capital consists of multiple steps. First we calculated the series of regional PPP (using 2000 as the base year) investmen en we used the Perpetual Inventory Method (as described before) to estimate the regional net capital stock.

Table 2 contains the data sources for variables used for estimating the TFP equation.

Table 2 – Data sources of the TFP equation

Variable Name	Description	Source
$TFP_{i,t}$	Total Factor Productivity	Authors' own calculations as described in section 3.1.1
$HUMCAP_{i,t}$	Population (aged 25-64) with tertiary education attainment by sex and NUTS2 regions (1000 capita)	Eurostat
$REDI_{i,t}$	Regional Entrepreneurship and development index	Authors' own calculations as described in section 3.1.3
$PATSCKR_{i,t}$	Patent stock calculated by the PIM using patent Total stocks of registered patents at regional level	Authors' own calculations using Eurostat patent data

Table 3 contains the data sources for variables used for estimating the patent equation.

Table 3 – Data sources of the patent equation

Variable Name	Description	Source
$PAT_{i,t}$	Number of patent registrations	Eurostat
$EMP_{i,t}$	Employment by sex, age and NUTS 2 regions	Eurostat
$RD\_TOTAL_{i,t}$	Total expenditures on research and development	Eurostat
$ENQFP_{i,t}$	Ego Network Quality index	Authors' elaboration on EU Framework Program Data
$PATSTCKN_{i,t}$	Total stocks of registered patents at country level	Authors' elaboration on Eurostat patent data.

3.1.5 Region-specific calibration of the parameters in the TFP block

After estimating the two equations of the TFP block (TFP equation and patent equation), we have a system of equations which is able to simulate the effects of different interventions affecting research and development, human capita, networking or the entrepreneurial climate on regional TFP. One drawback of this system is that the estimated coefficients which drive these impacts are common across all regions in the model, reflecting average tendencies in the sample of regions. However, one may argue that due to the large differences in the development level of European regions, mechanisms through which different interventions affect regional productivities differ largely across regions.

We control for these differences in two ways:

- First, in both equations the interaction terms render the respective coefficients of R&D, human capital, network quality and entrepreneurship development level regions-specific.
- Second, we augment this heterogeneity with a specific calibration process through which region-specific parameters are calculated through an optimization process to improve model fit. This second method is discussed briefly in what follows.

Given the observed data listed in Tables 2 and 3, we fit linear trends on these data points for all variables, except regional and national patent stocks (the former is directly given by equation (T21) and the latter is calculated by summing up regional patent stocks in each period). After trend fittings, we extrapolate the trend for out-of-sample years. These trends constitute the baseline of the TFP block.

After having the extrapolated trend values for all variables in the TFP block (except regional and national patent stocks which are calculated in the baseline according to equations (T21) and country-level aggregation), we run the regressions in (T20) and (T21) on these data points as well. Coefficients estimated on the historical data and coefficients estimated on the trend data stay fairly close to each other, therefore the mechanisms governing regional patent creation and total factor productivity are reasonably well replicated with the trend data based estimations.

The coefficients estimated on the trend data constitute the basis of region-specific parameter calibrations in the next step. The aim of the calibration is to find region-specific values for selected parameters, which improve the overall fit of the model while they meet certain conditions. After a careful selection procedure among several model versions three coefficients of the TFP block, namely the constant term and the coefficient of employment in the patent equation (parameters α and β_2 in equation T21) and the constant term in the TFP equation (parameter α in equation T20) are calibrated. This results in an optimization procedure where the objective function is the sum of the following five elements:

- Mean average percentage error of the regional patent application variable (average percentage deviation of simulated $PAT_{i,t}$ values from the trend values).
- Mean average percentage error of the TFP variable (average percentage deviation of simulated $TFP_{i,t}$ values from the trend values).
- Mean average percentage error of the average calibrated region-specific constant terms in the patent equation (average percentage deviation of calibrated constant terms from the trend-based estimated values).
- Mean average percentage error of the average calibrated region-specific coefficient of employment in the patent equation (average percentage deviation of calibrated coefficients from the trend-based estimated values).
- Mean average percentage error of the average calibrated region-specific constant terms in the TFP equation (average percentage deviation of calibrated constant terms from the trend-based estimated values).

As a result of this calibration process, we end up with region-specific parameter values for the listed three parameters of the TFP block which improve the fit of the TFP block equations and retain the average tendencies represented by the trend-based estimation. This way we obtain region-specific mechanisms built in the TFP block with respect to effects of exogenous variables on patenting activity and the productivity of the regions.

The final TFP block thus constitutes of equations (T20) and (T21) together with the aggregation of regional patent stocks to national patent stocks, where the coefficients and constants are either econometrically estimated on the out-of-sample trend values (being universal across regions) or calibrated according to the previous method (being region-specific). Table 4 summarizes the coefficients of the TFP block.

Table 4 – Coefficients of the TFP block and their calculation

Coefficient	Method of derivation	Level of specification
	TFP equation (T20)	
α	Calibration to improve model fit on the extrapolation period, starting from estimated values	Region-specific (heterogeneous)
β_1	Econometric estimation on out-of-sample extrapolated trend values	Universal (homogenous)
β_2	Econometric estimation on out-of-sample extrapolated trend values	Universal (homogenous)
β_3	Econometric estimation on out-of-sample extrapolated trend values	Universal (homogenous)
	Patent equation (T21)	
α	Calibration to improve model fit on the extrapolation period, starting from estimated values	Region-specific (heterogeneous)
β_1	Econometric estimation on out-of-sample extrapolated trend values	Universal (homogenous)
β_2	Calibration to improve model fit on the extrapolation period, starting from estimated values	Region-specific (heterogeneous)
β_3	Econometric estimation on out-of-sample extrapolated trend values	Universal (homogenous)
β_4	Econometric estimation on out-of-sample extrapolated trend values	Universal (homogenous)

3.2 The SCGE model block

The SCGE model block in the GMR approach serves to integrate spatial issues in the model. Spatial Computable Equilibrium (SCGE) models add the spatial dimension to the (usually spaceless) CGE models. This first means that the number of spatial units is larger than one. The term spatial units in SCGE models denotes subnational region[illegible]dditional extension to CGE models that the regions are interconnected by trade linkages and migration, transportation costs are explicitly accounted for and (positive and negative) agglomeration effects are also parts of the model structures.

Features of GMR models are usually determined by data availability to a large extent. At the regional level data are usually not as ~~much~~ detailed as at the national level and the modeler should adjust to this situation. The model distinguishes between short run and long run equilibriums. In short run equilibrium each region is in equilibrium in all the regional markets. However this does not mean that the whole regional system is in equilibrium. In case utilities differ across regions the whole system is not in equilibrium. Utility differences will induce labor migration (followed by the migration of capital). In the long run migration tends to the state where the system reaches the equilibrium state where interregional utility differences disappear.

In what follows, we provide a brief discussion of the setup of the SCGE model block as well as the dataset used for its calibration.

3.2.1 Equations in the SCGE model block and their calibration

3.2.1.1 The supply side

The SCGE model, harmonized with the QUEST III MACRO model operates with increasing returns, monopolistic competition characterized with markup pricing. The basic equation of the model is the Cobb-Douglas production function which determines output (Y) using labor (L) and capital inputs. The two capital inputs are private capital (K).

$$Y_{i,t} = \tilde{A}_{i,t} \cdot L_{i,t}^{\delta+\gamma} \cdot K_{i,t}^{\beta} \tag{S1}$$

where $\delta + \gamma, \beta$ and are the respective production elasticities of the production factors. The setup includes a special element, L^{γ} which captures agglomeration externalities in line with the assumptions in the TFP block. δ is therefore the "standard" production elasticity of labor and γ measures the agglomeration effect. The production elasticities are parameters in the SCGE block and their values are given by the estimations described previously in section 3.1.1 with the addition that national elasticities were adjusted to obtain region-specific production elasticities in order to fit the data. i stands for regions and t stands for time periods. The C-D production function in this setting is characterized by increasing returns to scale thus $(\delta + \gamma + \beta) = (\alpha + \beta) > 1$, where $\alpha = \delta + \gamma$.

$\tilde{A}_{i,t}$ plays a crucial role in the system as the SCGE model gets its TFP shocks from the TFP model via this variable. Due to the agglomeration economies implemented in the production function, the following relationship exists:

$$\tilde{A}_{i,t} = \frac{TFP_{i,t}}{L^{\gamma}{}_{i,t}} \tag{S2}$$

where the numerator gets its actual value in the simulations according to the shocks to research, human capital and networking.

In line with the MACRO block, we assume monopolistic competietion and markup pricing in the model. Markup pricing is characterized according to the following equations. Marginal cost is (leaving out the time and region subscripts for simplicity):

$$MC = \frac{dTC}{dY} = \frac{w^{\frac{\alpha}{\alpha+\beta}} \cdot r^{\frac{\beta}{\alpha+\beta}}}{\tilde{A}^{\frac{1}{\alpha+\beta}} \cdot \alpha^{\frac{\alpha}{\alpha+\beta}} \cdot \beta^{\frac{\beta}{\alpha+\beta}}} \cdot Y^{\frac{1-\alpha-\beta}{\alpha+\beta}} \tag{S3}$$

Average cost is:

$$AC = \frac{TC}{Y} = (\alpha + \beta)\frac{w^{\frac{\alpha}{\alpha+\beta}} \cdot r^{\frac{\beta}{\alpha+\beta}}}{\tilde{A}^{\frac{1}{\alpha+\beta}} \cdot \alpha^{\frac{\alpha}{\alpha+\beta}} \cdot \beta^{\frac{\beta}{\alpha+\beta}}} \cdot Y^{\frac{1-\alpha-\beta}{\alpha+\beta}} = (\alpha + \beta) \cdot MC \tag{S4}$$

In monopolistic competition price equals average cost:

$$q = \frac{\varepsilon}{\varepsilon - 1} MC \tag{S5}$$

where $\varepsilon/(\varepsilon - 1)$ is the markup and q is the producer price. It can be proven that $\alpha + \beta = \varepsilon/(\varepsilon - 1)$ where ε equals to the elasticity of substitution that is applied in the MACRO model.

Labor demand can be written:

$$L^{dem} = \left(\frac{Y}{\tilde{A}}\right)^{\frac{1}{\alpha+\beta}} \cdot \left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\alpha+\beta}} \tag{S6}$$

where r is the price of capital and w is the price of labor. The demand for capital is similarily:

$$K^{dem} = \left(\frac{Y}{\tilde{A}}\right)^{\frac{1}{\alpha+\beta}} \cdot \left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \tag{S7}$$

The demand for output is given by:

$$X = \frac{Z}{p} \tag{S8}$$

where Z is income spent $(Z = wL + rK)$ which w and r stand for wage and capital rent. Where p is the purchaser price level in a region. Note, that producer prices of teh firms in a region differ from the purchaser prices of commodities in the same region due to interregional trade (see later).

3.2.1.2 The demand side

We assume that household preferences are homogenous and described by the following utility function:

$$U_{i,t} = \bar{\alpha}_H \ln\left(\frac{H_{i,t}}{N_{i,t}}\right) + \bar{\beta}_H \ln x_{i,t} \tag{S9}$$

where $x_{i,t}$ stands for consumption, $H_{i,t}$ for housing, while $\bar{\alpha}_H$ and $\bar{\beta}_H$ are paramters. The latter parameters are calibrated as in Table 4.

Households' individual budget is formulated as

$$w_{i,t}\frac{L_{i,t}}{N_{i,t}} + r_{i,t}\frac{K_{i,t}}{N_{i,t}} = p_{i,t}x_{i,t} \tag{S10}$$

where $N_{i,t}$ is regional population and $p_{i,t}$ is the general level of prices in the region. Maximizing utility in (S9) under (S10) leads to the demand function for goods:

$$X_{i,t} = \frac{\overline{\beta}_H}{1-\alpha_H}\frac{1}{p_{i,t}}\left(w_{i,t}\frac{L_{i,t}}{N_{i,t}} + r_{i,t}\frac{K_{i,t}}{N_{i,t}}\right)N_{i,t} \qquad \text{(S11)}$$

Some of the goods are produced in the region but some of them are shipped from other regions. We define $s_{i,j,t}$ as the share of region i in the market of region j. Assuming iceberg transportation costs the following CES demand function is derived:

$$s_{i,j,t} = \gamma_i\left[\frac{(1+\tau_{i,j})q_{i,t}}{p_{j,t}}\right]^{-\mu} \qquad \text{(S12)}$$

where μ is the elasticity parameter of the CES function, γ_i is the share parameter and $\tau_{i,j}$ is the cost of transportation from region i to region j.

The general price level, $p_{j,t}$ is calculated as follows:

$$p_{j,t} = \sum_i s_{i,j,t}q_{i,t}(1+\tau_{i,j}) \qquad \text{(S13)}$$

$$p_{j,t} = \left\{\sum_i \gamma_i[(1+\tau_{i,j})q_{i,t}]^{1-\mu}\right\}^{\frac{1}{1-\mu}} \qquad \text{(S14)}$$

3.2.1.3 Short run equilibrium conditions

Factor market clearing on the regional level we can write:

$$L_{i,t}^{sup} = L_{i,t}^{dem} \qquad \text{(S15)}$$

$$K_{i,t}^{sup} = K_{i,t}^{dem} \qquad \text{(S16)}$$

The model actually recalculates factor prices $w_{i,t}$ and $r_{i,t}$ until the equilibrium conditions hold.

In our model the average interest rate serves as the numeraire

$$\bar{r} = \frac{\sum_i r_{i,t}K_{i,t}^{(sup)}}{\sum_i K_{i,t}^{(sup)}} = const \qquad \text{(S17)}$$

Demand for goods produced in region i is $X_{i,t}$ while the supply is $Y_{i,t}$. Taking into account transportation cost the equilibrium conditions in the goods market is given by:

$$X_{j,t} = \sum_j s_{i,j,t}Y_{i,t}(1+\tau_{i,j}) \qquad \text{(S18)}$$

3.2.1.4 Modeling migration

Equilibrium conditions given above provide a one-time equilibrium in across regions: they determine an optimal allocation of goods across regions, given the supply of production factors.

As a next step, regarded as „long run” in the SCGE model block, interregional differences in utilities result in labor migration thus changing labor supplies in the consecutive period:

$$L'_{i,t} = L_{i,t} + LMIGR_{i,t} \qquad \text{(S19)}$$

Labor migration ($LMIGR_{i,t}$) is given by the following equation:

$$LMIGR_{i,t} = \Phi \left(e^{\Theta(U_{i,t}^{*}+c_{i,t})} - e^{\Theta \cdot AVG(U_{i,t}^{*}+c_{i,t})} \right) L_{i,t} \qquad \text{(S20)}$$

where $U_{i,t}^{*}$ is regional utility, c_i is a region-specific const Φ and Θ determine the speed of migration. AVG stands for weighted averaging utilities where employment is the weigh

3.2.1.5 The calibration of the parameters of the SCGE block

Table 5 below provides details on the calibration of different model parameters in the SCGE block. Some parameters are used as given from other estimations and some are linked to the data by calibrating them in the baseline version.

Table 5 – Parameters of the SCGE model

Parameter	Source
β	Calibrated from primary income shares
δ	Calibrated from primary income shares
γ	Estimated econometrically – from the TFP block (see Appendix A.1)
α	$\alpha = \gamma + \delta$
ε	$\alpha + \beta = \varepsilon/(\varepsilon - 1)$ according to the relationship in the MACRO model
$\bar{\alpha}_H$	Set at 0.1.
$\bar{\beta}_H$	Set at 0.9.
γ_i	Calibrated: in the baseline the algorithm searches for the value when the model produces the values of all the variables which are equal to the respective observed values.
$\tau_{i,j}$	Calculated based on transportation costs.
μ	Set at 2.439
ϕ	Set at 1.000
θ	Set at 0.010
$c_{i,t}$	Calibrated: in the baseline the algorithm searches for the value which produces zero migration in the baseline. Scenarios thus reflect additional migration resulting from the interventions.

3.2.2 The SCGE block database

Table 6 contains the data sources for the calibration of the SCGE model.

Table 6 – Data bases for variables in the SCGE model block

Variable Name	Description	Source
Y	Regional Gross Value Added	Eurostat
L	Employment	Eurostat
K	Regional Capital Stocks	Calculated, using PIM
w	Wages	Eurostat
r	Capital rent	Numeraire, calculated by the model
H	Housing Stocks	Eurostat
N	Population	Eurostat

3.2.3. Consistency adjustments between the SCGE and MACRO model blocks

The MACRO model block in GMR-Europe (described in the next section) is capable of calculating the dynamic macro path of the main variables. This dynamic aspect however is missing in the SCGE block which is responsible for the calculation of the spatial distribution of economic activities. Without the interaction of the two blocks the SCGE sub-model is not capable of calculating those dynamic paths. Thus, dynamics comes from the macro block in the form of macro value added, employment, capital and investment. In the base year macro variables are perfectly consistent with aggregated regional values. After the first time period dynamic changes in macro variables are needed to be distributed to individual regions in a theoretically consistent way, without creating distortions in the model's predictions. Our solution consists of three steps: 1) the adjustment of regional employment, 2) the adjustment of regional investment, 3) the adjustment of the regional value added and capital. Employment and investment are adjusted independently, we calculate the absolute difference between the macro and aggregated regional values and redistribute them on the basis of productivity changes and the economic size of regions. This means that those regions will be rewarded by bigger share of investment that experienced higher productivity change and bigger in size. Since value added, employment and capital are interconnected in the production function we cannot adjust all of them independently. If we adjust the level of employment and capital the level of value added will be given by the production function. Thus, we chose to eliminate inconsistencies in the case of those variables (investment, employment and value added) that are actually known from data and we allow for some level of inconsistency in case of capital which is estimated. Therefore, the consistent level of value added is generated by the adjustment of regional capital stocks (through the production function) where we do not prescribe full consistency between the macro and regional level. A more detailed account of these adjustment/distribution processes are provided in appendix A.3.

3.3 The MACRO model block

The macroeconomic block of GMR is given by a standard, large-scale DSGE (dynamic, stochastic, general equilibrium) model. The role of this model block is to model dynamic economic effects and to provide a framework for the static SCGE block with the dynamics of necessary macro variables. The macroeocnomic model we use is the QUEST III model developed by the European Commission for the Euro area, and was reestimated on data for the Eurozone and additional countries in the GMR-Europe model. The description of the original model can be found in Ratto et al. (2009).

Modern macroeconomic analysis builds on general equilibrium models which consider market equilibrium as a gravitational point of the economy. These models started to penetrate mainstream macroeconomics as an answer to the Lucas critique which draws the attention to the fact that the efficiency of policy interventions can be counteracted by mechanisms driven by the modified decisions of rational actors expecting these interventions. This critique proved to be a significant theoretical challenge for Keynesian macroeconometric models which, as a result of their inherent structure, can not account for these adjustments. The answer to these challenges were basically theory-based, and micro-founded structural models which, as a result of their former characteristics, are able to explicitly handle the effects resulting from the change in economic actors' behavior.

The general equilibrium paradigm entered mainstream macroeconomics with RBC (real business cycle) models, which provide a supply-side (basically productivity-based) explanation for business cycles. These models, although, robust to the Lucas critique, are less able to explain that empirical evidence that demand-side shocks have persistent real effects. Subsequent (also called new

Keynesian) model developments tried to make the models more realistic by including market imperfections (mainly monopolistic competition) and other frictions (adjustment costs, rigid prices, non-optimizing actors).

Building on these veins of the literature, in the last two decades a kind of synthesis has been established in modern macroeconomics which retains general equilibrium as a sound theoretical basis which drives long run dynamics in the economy, but in the short run the just mentioned frictions and imperfections can generate even large deviations from this long run equilibrium path. During this period DSGE (dynamic stochastic general equilibrium models) step forward as a workhorse of macroeconomics. These models are dynamic because they explicitly take into account intertemporal decisions of economic actors; they are stochastic as the structural relationship and variables of the model can be hit by different shocks driving the economy away from the equilibrium path; they are general equilibrium as they assume market clearing (even if markets are not perfect).

Although DSGE models provide the advantage of explicit microeconomic background and theoretical coherence in contrast to traditional macroeconometric models, partly as a consequence of these characteristics, their empirical fit to the data is problematic as the models do not capture the data-generating process behind observed time series. In spite of this, important development has been done in respect: Smets and Wouters (2003) for example show that a DSGE model based on new Keynesian background can forecast macro time series as precisely as an empirical VAR model.

In the typical DSGE models households decide on consumption, investment and supply differentiated labor, leading to a wage setting power on their side. This labor is employed by the firms, they rent capital and supply differentiated goods to households on a monopolistically competitive market, leading to a price setting power on their side. Both households and firms make decisions in a dynamic environment, maximizing the present value of future utility and profits, through setting the above variables. A basic characteristic of DSGE models is that actors form rational expectations with regards to the future.

Both households and firms face nominal rigidities (rigid prices and wages, indexing) which constrain their wage and price setting power. Capital accumulates endogenously in these models, but investment and capacity utilization is subject to adjustment costs. The preferences of households generally contain habit formation, so that utility is not only dependent on current but also on past consumption (with a specific weight). Most of these models operate with a limited fiscal policy block, and monetary policy is generally integrated through an interest rate (Taylor) rule. This basic structure is then augmented by different shocks which affect the supply side (productivity, labor supply), the demand side (preferences, government expenditures), costs (price- and wage markup, risk premium) or the monetary rule. These shocks are modeled as first order autoregressive processes most of the time. (Tovar, 2008)

The popularity of DSGE models are signaled by the fact that many central bank and economic analyst institute use these models for policy impact analysis or forecasting. Just to mention some: the Federal Reserve in the US (Erceg et al., 2006), the European Central Bank in the Eurozone (Christoffel et al., 2008), the Bank of England in Great Britain (Harrison et al., 2005), or the Hungarian Central Bank (Jakab and Világi, 2008; Szilágyi et al., 2013).

3.3.1 Equations of the MACRO model block

The macroeconomic block of the GMR model is a standard DSGE model which describes the relationship of for macroeconomic sectors (households, firms, government, foreign sector). It uses 104 endogenous variables to describe this structure and the dynamics are driven by 23 exogenous shock variables.[7] The model equations are determined by 120 structural parameters, and the standard deviations of the 23 shocks also appear as parameters. In what follows, we describe the equations describing each sectors in detail.

Those equations which are finally used in the model are basically defined in growth rates and shares/ratios to the GDP. However, during the derivations, we use levels instead of rates in order to help the understanding. Where appropriate, we move to the declaration system of the technical equations in rates. Due to the many equations and different derivations, we split the numbering of equations into two parts. We use letter 'A' to denote equations which are presented only as additional, guiding relationships in the derivations, whereas the letter 'M' is used to denote those equations which constitute the final, estimated model.

3.3.1.1 The households

A typical tool of mainstream DSGE models, primarily to indicate real effect of fiscal interventions, is to split the household sector into two parts, namely the 'ricardian' and 'non-ricardian' or in other words non-liquidity constrained and liquidity constrained households. While the former have unconstrained access to financial markets, can borrow and save part of their income, the latter spend their current income solely to consumption.

Ricardian households

The ricardian households of the model are characterized by the following utility function, which defines utility in function of consumption and leisure. Both factors are equipped with habit formation and we also define preference shocks.

$$U_t^R(C_t^R, L_t^R) = \frac{\exp(u_t^C)\left[\left(C_t^R - h^C C_{t-1}^R\right)\left(1 - \exp(u_t^L)\omega\left(L_t^R - h^L L_{t-1}^R\right)^\kappa\right)\right]^{1-\sigma^C}}{1-\sigma^C} \qquad \text{(A1)}$$

In te above utility function C_t^R denotes the consumption of the representative ricardian household in period t, L_t^R is the labor supply of the household in period t, u_t^C and u_t^L are exogenous shocks to preferences, h^C and h^L are the habit parameters, σ^C, κ and ω are further preference parameters. The partial derivative of the above utility function according to consumption (C_t^R) is:

$$UC_t^R = \exp(u_t^C)(C_t^R - h^C C_{t-1}^R)^{-\sigma^C}(1 - \exp(u_t^L)\omega(L_t^R - h^L L_{t-1}^R)^\kappa)^{1-\sigma^C} \qquad \text{(A2)}$$

The partial derivative according to leisure $(1 - L_t^R)$ is:

[7] The original model specification estimated for the Eurozone uses 19 exogenous shocks which were augmented by four further effects in order to fit the model into the specific framework of the GMR model.

$$UL_t^R = \exp(u_t^C)(C_t^R - h^C C_{t-1}^R)^{1-\sigma^C}(1-\exp(u_t^L)\omega(L_t^R - h^L L_{t-1}^R)^{\kappa})^{-\sigma^C}\exp(u_t^L)\omega\kappa(L_t^R - h^L L_{t-1}^R)^{\kappa-1} \quad \text{(A3)}$$

The two relationships above are modified as the model operates with growth rates and shares to GDP. Let's multiply equations (A2) and (A3) both with $[(P_t^C/(Y_t P_t)/(1+\overline{GY})]^{-\sigma^C}$, where Y_t stands for GDP, P_t^C is the price level of consumption goods, P_t is the price level of GDP (the GDP deflator), and $\overline{GY}$ is the steady state growth rate of GDP (which is a parameter of the model).

$$NUC_t^R = UC_t^R\left(\frac{P_t^C}{Y_t P_t(1+\overline{GY})}\right)^{-\sigma^C} \quad \text{(A4)}$$

$$NUL_t^R = UL_t^R\left(\frac{P_t^C}{Y_t P_t(1+\overline{GY})}\right)^{-\sigma^C} \quad \text{(A5)}$$

The two values above define the respective marginal utilities compared to GDP on a nominal basis (utility is monetized on the price level of consumption goods). Substituting the respective marginal utilities into (A4) and (A5):

$$NUC_t^R = \exp(u_t^C)\left(\frac{C_t^R P_t^C}{Y_t P_t(1+\overline{GY})} - h^C\frac{C_{t-1}^R P_t^C}{Y_t P_t(1+\overline{GY})}\right)^{-\sigma^C}(1-\exp(u_t^L)\omega(L_t^R - h^L L_{t-1}^R)^{\kappa})^{1-\sigma^C} \quad \text{(A6)}$$

$$NUL_t^R = \exp(u_t^C)\left(\frac{C_t^R P_t^C}{Y_t P_t(1+\overline{GY})} - h^C\frac{C_{t-1}^R P_t^C}{Y_t P_t(1+\overline{GY})}\right)^{1-\sigma^C}(1-\exp(u_t^L)\omega(L_t^R - h^L L_{t-1}^R)^{\kappa})^{-\sigma^C}\exp(u_t^L)\omega\kappa(L_t^R - h^L L_{t-1}^R)^{\kappa-1} \quad \text{(A7)}$$

Let's introduce the following notation: $CSN_t^R = (C_t^R P_t^C)/(Y_t P_t)$, which is simply the ratio of ricardian households' nominal consumption to nominal GDP. Using this definition, (A4) and (A5) can be written in the following form which are at the same time the first equations of the model used in estimation and simulation:

$$NUC_t^R = \exp(u_t^C)\left[CSN_t^R\left(1 - h^C\frac{1}{1+GC_t^R-\overline{GY}}\right)\right]^{-\sigma^C}(1-\exp(u_t^L)\omega(L_t^R - h^L L_{t-1}^R)^{\kappa})^{1-\sigma^C} \quad \text{(M1)}$$

$$NUL_t^R = \exp(u_t^C)\left[CSN_t^R\left(1 - h^C\frac{1}{1+GC_t^R-\overline{GY}}\right)\right]^{1-\sigma^C}(1-\exp(u_t^L)\omega(L_t^R - h^L L_{t-1}^R)^{\kappa})^{-\sigma^C}\exp(u_t^L)\omega\kappa(L_t^R - h^L L_{t-1}^R)^{\kappa-1} \quad \text{(M2)}$$

where $GC_t^R = C_t^R/C_{t-1}^R - 1$, is the growth rate of real consumption in the case of ricardian households. On the basis of equations (M1) and (M2), together with equations (A4) and (A5) define the growth rate of the marginal utility of consumption (in absolute and real terms):

$$\log(NUC_t^R) - \log(NUC_{t-1}^R) = GUC_t + \sigma^C(GY_t - \overline{GY} + \pi_t - \pi_t^C) \quad \text{(M3)}$$

where $GUC_t = UC_t^R/UC_{t-1}^R - 1$ denotes the rate of change in the marginal utility of consumption, $GY_t = Y_t/Y_{t-1} - 1$ is the growth rate of per capita GDP, π_t is inflation rate (based on the GDP deflator), and π_t^C is the rate of change in the price of consumption goods.

Ricardian households spend their income, over consumption, on investment in physical capital, domestic and foreign bonds, while keeping the remaining income in money. Their budget constraint, written in nominal terms is as follows:

$$(1 + t^C)P_t^C C_t^R + P_t^I I_t + M_t + NB_t + E_t NB_t^F = M_{t-1} + (1 + i_{t-1})NB_{t-1} + (1 + i_{t-1}^F)\left(1 - rf\frac{E_t NB_{t-1}^F}{P_{t-1}Y_{t-1}} + u_t^F\right)NB_{t-1}^F + [i_{t-1}^K - rp_t - t^P(i_{t-1}^K - rp_t + \delta)]P_{t-1}^I K_{t-1} + (1 - t_t^W - ssc)W_t L_t^R - \frac{\gamma_W L_t^R}{2}\frac{(\Delta W_t)^2}{W_{t-1}} + PR_t P_t \quad \text{(A8)}$$

The expenditure (left-hand) side of this budget constraint sums (respectively) consumption, investment in physical capital, money holding, domestic and foreign bonds and lump sum taxes. t^C is the rate of consumption tax (a parameter of the model), M_t is money supply, NB_t is the domestic and NB_t^F is the foreign nominal stock of bonds and E_t is the nominal exchange rate. On the revenue side t^P is the tax rate on capital income, , i_t is the domestic and i_t^F is the foreign interest rates on bonds, i_t^K is the nominal return on physical capital. rp_t is the risk premium on physical capital investment, δ is the depreciation rate, t_t^W is the rate of labor income tax, ssc is the rate of social security contributions, $W_t$is the nominal wage, while PR_t is the (real) profit income. There are two non-trivial elements on the right hand side. First, risk premium on foreign bonds, which is a function of foreign debt (the effect of external debt on this elemen is given by parameter rf) and an exogenous shock (u_t^F). Second, there is an adjustment cost coming from changes in the wage (more details are given in the section on wage setting), which depends on the employment level and wage change (ΔW_t), while its strength is determined by parameter γ_W.

The decision of ricardian households are also influenced by installations costs linked to physical capital investments: only a part of the total amount of purchasing power spent on physical capital investment (denoted by I_t) is in effect installed as physical capital (J_t), the difference melted in installation costs. This relationship is defined in the following equation:

$$I_t = J_t\left(1 + \frac{\gamma_K}{2}\left(\frac{J_t}{K_t}\right)\right) + \frac{\gamma_I}{2}(\Delta J_t)^2 \quad \text{(A9)}$$

where γ_K and γ_I are parameters determining installation costs. As a result, the accumulation of physical capital is described by the following formula:

$$K_t = J_t + (1 - \delta)K_{t-1} \quad \text{(A10)}$$

The decision problem of the households is to maximize (A1) on an infinite time horizon subject to the budget constraint (A8) and further constraints (A9) and (A10). The five decision variables of the household are consumption (C_t^R), purchases of domestic and a foreign bonds (NB_t and NB_t^F), investment in physical capital, (I_t), and the planned level of physical capital (K_t).

Using the (A8) budget constraint in real terms (dividing through by P_t) we obtain the following first order conditions with respect to consumption and domestic bonds respectively (we omit the expectations operator for the sake of clarity):

$$UC_t^R - \lambda_t \frac{(1+t^C)P_t^C}{P_t} = 0 \tag{A11}$$

$$-\lambda_t + \lambda_{t+1}\beta(1+i_t)\frac{P_t}{P_{t+1}} = 0 \tag{A12}$$

where λ_t is the Lagrange-multiplier of the budget constraint. Eliminating λ_t from these two equations we get

$$\frac{1}{\beta} = \frac{UC_{t+1}^R}{UC_t^R}(1+i_t)\frac{P_t^C}{P_{t+1}^C} \tag{A13}$$

which, after taking logarythms, we obtain the (approximate) form of the Euler equation:

$$\frac{1}{\beta} - 1 = GUC_{t+1} + i_t - \pi_{t+1}^C \tag{M4}$$

The first order condition with respect to foreign bonds in the decision problem of households is:

$$-\lambda_t + \lambda_{t+1}\beta(1+i_t^F)\left(1 - rf\frac{E_t NB_t^F}{P_t Y_t} + u_t^F\right)\frac{P_t}{P_{t+1}}\frac{E_{t+1}}{E_t} = 0 \tag{A13}$$

Using (A12) and (A13) we end up with uncovered interest rate parity

$$\frac{1+i_t}{1+i_t^F}\left(1 - rf\frac{E_t NB_t^F}{P_t Y_t} + u_t^F\right) = \frac{E_{t+1}}{E_t} \tag{A14}$$

Loglinearizing equation (A14) gives the approximate form of uncovered interest rate parity which is directly used by the model:

$$i_t = i_t^F + GE_{t+1} - rf \cdot B_t^F + u_t^F \tag{M5}$$

where GE_t is the growth rate of the nominal exchange rate while B_t^F is the external debt to GDP ratio (in nominal terms - $B_t^F = E_t NB_t^F / P_t Y_t$).

In the optimization problem the partial derivatives with respect to investment and physical capital lead to the following first order conditions respectively:

$$-\xi_t + \xi_t\beta(1-\delta) + \lambda_{t+1}\beta[(1-t^P)(i_t^K - rp_t) + t^P\delta]\frac{P_t^I}{P_{t+1}} = 0 \tag{A15}$$

$$-\lambda_t\frac{P_{t-1}^I}{P_t}\left(1 + \gamma_K\frac{J_t}{K_t} + \gamma_I \Delta J_t\right) - \lambda_{t+1}\beta\frac{P_t^I}{P_{t+1}}\gamma_I \Delta J_{t+1} + \xi_t = 0 \tag{A16}$$

where ξ_t is the Lagrange-multiplier of the capital accumulation equation (A10) (as an optimization constraint), whereas equation (A9) as a constraint is substituted into equation (A8). Define the present value of the return on physical capital (Tobin-Q) as

$$Q_t = \frac{\xi_t}{\lambda_t}\frac{P_t}{P_t^I} \qquad \text{(A17)}$$

Using equations (A15)-(A17), and the relationship for λ_{t+1}/λ_t given by first order condition (A12), the following two equations are obtained as drivers of households' investment decisions:

$$\gamma_K \frac{J_t}{K_t} + \gamma_I \Delta J_t - \frac{\gamma_I \Delta J_{t+1}}{1+i_t} = Q_t - 1 \qquad \text{(A18)}$$

$$Q_t = \frac{1-\delta}{1+i_t}\frac{P_{t+1}^I}{P_t^I} + \frac{(1-t^P)(i_t^K - rp_t) + t^P\delta}{1+i_t} \qquad \text{(A19)}$$

Equation (A18) gives investments in function of Q_t. Introduce GI_t for the growth rate of investment and IK_t which denotes the ratio of investment to per capita capital stock. Using these definitions, equation (A18) can be written alternatively as

$$\gamma_K[IK_t - (\delta + \overline{GY} + g^{UI} + g^{pop})] + \gamma_I[GI_t - (\overline{GY} + g^{UI})] - \frac{\gamma_I}{1+i_t}[GI_{t+1} - (\overline{GY} + g^{UI})] = Q_t - 1 \qquad \text{(M6)}$$

In the above equation $\overline{GY}$ is the steady state growth rate of GDP, g^{UI} is the steady state growth rate of the productivity of intermediate goods and g^{pop} is the growth rate of population, which values are the parameters of the model. The difference in (M6) compared to (A18) is that investment growth and investment to capital stock ratio is written in their deviations from steady state. In subsequent parts of this description we show that the growth rate of investment in steady state is $(\overline{GY} + g^{UI})$, and the ratio of investment to capital stock per capita in the steady state is $(\delta + \overline{GY} + g^{UI})$ which is adjusted to the population growth because equations (A18) and (M6) use total capital stock levels.[8]

Liquidity constrained households

The utility function of non-ricardian households does not contain habit formation in consumption and preference shock to consumption, but apart from these, it is similar to the utility function of the ricardian households:

$$U_t^{NR}(C_t^R, L_t^R) = \frac{\left[C_t^R\left(1 - \exp(u_t^L)\omega(L_t^R - h^L L_{t-1}^R)^\kappa\right)\right]^{1-\sigma^C}}{1-\sigma^C} \qquad \text{(A20)}$$

Using the same method as for the ricardian households, we obtain the marginal utilities analogous to those in (M1) and (M2):

$$NUC_t^{NR} = (CSN_t^R)^{-\sigma^C}(1 - \exp(u_t^L)\omega(L_t^R - h^L L_{t-1}^R)^\kappa)^{1-\sigma^C} \qquad \text{(M7)}$$

$$NUL_t^R = (CSN_t^R)^{-\sigma^C}(1 - \exp(u_t^L)\omega(L_t^R - h^L L_{t-1}^R)^\kappa)^{-\sigma^C}\exp(u_t^L)\omega\kappa(L_t^R - h^L L_{t-1}^R)^{\kappa-1} \qquad \text{(M8)}$$

Liquidity constrained households do not optimize, their behavior is described by their budget constraint, which is:

[8] To define steady state we need per capita variables because these can be constant when population changes.

$$(1+t^C)P_t^C C_t^{NR} + T_t^{LS} P_t = (1-t_t^W - ssc)W_t L_t^{NR} + TR_t P_t \quad \text{(A21)}$$

where in addition to the previous notation C_t^{NR} is the consumption of non ricardian households, L_t^{NR} is their labor supply, T_t^{LS} is the real value of lump sum taxes and TR_t is the level of transfers.[9] Dividing through (A21) with $Y_t P_t$ we get:

$$(1+t^C)\frac{P_t^C C_t^{NR}}{Y_t P_t} + \frac{T_t^{LS}}{Y_t} = (1-t_t^W - ssc)\frac{W_t}{Y_t P_t} L_t^{NR} + \frac{TR_t}{Y_t} \quad \text{(A22)}$$

Define $CSN_t^{NR} = P_t^C C_t^{NR}/Y_t P_t$, which is the ratio of the nominal consumption of non ricardian households to nominal GDP, let $TRW_t = TR_t/Y_t$ be the transfers to GDP ratio and $YWR_t = Y_t/(W_t/P_t)$ be the ratio of GDP and real wage. Define then the nominal share of wages in GDP as:[10]

$$WS_t = L_t \frac{1}{YWR_t} \quad \text{(M9)}$$

With these definitions the budget constraint in (A22) can be written in the form:

$$(1+t^C)CSN_t^{NR} + TY_t^{LS} = (1-t_t^W - ssc)WS_t + TRW_t WS_t \quad \text{(M10)}$$

Aggregation of households

The aggregation of the consumption of ricardian and non ricardian households are given by the following relationship where slc is the share of liquidity constrained households (a parameter of the model):$CSN_t = slc \cdot CSN_t^{NR} + (1-slc) \cdot CSN_t^R$ (M11)

3.3.1.2 The firms

The model splits the firms' sector into two parts. Firms producing final consumption goods operate on a monopolistically competitive market and use capital and labor as input. The other sector of firms produce capital (investment) goods), operate on a perfectly competitive market and use domestic and imported final goods as inputs.

Final good producers

Final good producers operate on a monopolistically competitive market. Their production technology is described by the following production function:

$$Y_t^j = A_t^\alpha (L_t^j - LO_t^j)^\alpha (ucap_t^j K_t^j)^{1-\alpha} (K_t^G)^{1-\alpha_G} \quad \text{(A23)}$$

where Y_t^j is the output of producer j, α is the partial production elasticity of labor, A_t is labor productivity characteristic to the whole economy, L_t^j is the labor utilization of producer j, LO_t^j is the overhead labor, K_t^j is the stock of physical capital, $ucap_t^j$ is capacity utilization, K_t^G is the level of public (infrastructural) capital and α_G is the additive inverse of the production elasticity of public capital.

[9] In the model only liquidity constrained households receive transfers and pay lump sum taxes.
[10] In the model $L_t^R = L_t^{NR} = L_t$.

The demand for goods produced by the final producers is determined by a nested CES utility function. The elasticity of substitution between domestic and imported goods is σ^M and the elasticity of substitution between domestic goods is σ^d. All sectors (households, firms, government, foreign sector) have identical preferences so the following demand function can be written for the goods produced by firm j:

$$Y_t^j = \frac{1-s^M}{n}\left(\frac{P_t}{P_t^j}\right)^{\sigma^d}\left(\frac{P_t^C}{P_t}\right)^{\sigma^M}\left(C_t + C_t^G + I_t^G + I_t^{inp} + EX_t\right) \quad \text{(A24)}$$

where n is the number of final good producers, s^M is the share of domestic absorption, P_t^j is the price set by firm j, P_t is the aggregate price level, P_t^C is the price level of consumption goods and in the last parenthesis we have the consumption demand of households and government, the investment demand of the government, the input demand of capital good firms and the export demand, respectively.

The decision of the firms is constrained by three adjustment costs. They face these costs when changing labor utilization, prices and capacity utilization, defined by the following equations respectively:

$$aL(L_t^j) = W_t\left[L_t^j u_t^L + \frac{\gamma_L}{2}(L_t^j - L_{t-1}^j)^2\right] \quad \text{(A25)}$$

$$aP(P_t^j) = \frac{\gamma_P}{2}\frac{(P_t^j - P_{t-1}^j)^2}{P_{t-1}^j} \quad \text{(A26)}$$

$$aU(ucap_t^j) = P_t^I K_t^j\left[\gamma_{U1}(ucap_t^j - \overline{ucap}) + \frac{\gamma_{U2}}{2}(ucap_t^j - \overline{ucap})^2\right] \quad \text{(A27)}$$

where γ_L, γ_P, γ_{U1} and γ_{U2} are the parameters of the adjustment cost functions, u_t^L is an exogenous shock to the adjustment cost to labor and $\overline{ucap}$ is the steady state value of capacity utilization.

The profit function of the firm is:

$$PR_t^j = \frac{P_t^j Y_t^j}{P_t} - \frac{W_t L_t^j}{P_t} - \frac{i_t^K P_t^I K_t^j}{P_t} - \frac{1}{P_t}\left[aL(L_t^j) + aP(P_t^j) + aU(ucap_t^j)\right] \quad \text{(A28)}$$

The decision problem of the firms is to maximize profit function (A28) on an infinite time horizon subject to constraints (A23)-(A27). Define the Lagrange function as follows (using the real interest rate (r_t) for discounting):

$$V = \sum_{t=0}^{\infty}\frac{1}{(1+r_t)^t}PR_t^j + \eta_t^j\left[Y_t^j - A_t^\alpha(L_t^j - LO_t^j)^\alpha(ucap_t^j K_t^j)^{1-\alpha}(K_t^G)^{1-\alpha_G}\right] \quad \text{(A29)}$$

then substitute the constraints (A24)-(A27) into the Lagrange function (A29). Differentiating the resulting optimization problem with respect to labor utilization L_t^j, we obtain the following first order condition:

$$-\frac{W_t}{P_t^j} + \alpha\eta_t^j\frac{Y_t^j}{L_t^j - LO_t^j} - \frac{W_t}{P_t^j}u_t^L - \frac{W_t}{P_t^j}\gamma_L(L_t^j - L_{t-1}^j) + \frac{W_{t+1}}{P_{t+1}^j}\frac{\gamma_L}{1+r_t}(L_{t+1}^j - L_t^j) = 0 \quad \text{(A30)}$$

Using the notation $YWR_t = Y_t/(W_t/P_t)$ defined previously and the fact that due to the symmetry of the monopolistic competition we can leave superscript j, equation (A30) can be written in the following form:

$$\frac{1+u_t^W}{YWR_t} = \eta_t \alpha \frac{1+lol_t}{L_t} - \frac{1}{YWR_t}\gamma_L(L_t - L_{t-1}) + \frac{1}{YWR_t}(1 + GY_t - \overline{GY})\frac{\gamma_L}{1+r_t}(L_{t+1} - L_t) \tag{M12}$$

Differentiating with respect to capacity utilization results in the next first order condition:

$$(1-\alpha)\eta_t^j \frac{Y_t^j}{K_t^j} - \frac{P_t^I}{P_t}\left[\gamma_{U1} + \gamma_{U2}\left(ucap_t^j - \overline{ucap}\right)\right] = 0 \tag{A31}$$

Introduce $KSN_t = (P_t^I K_t)/(P_t Y_t)$ which is the physical capital to GDP ratio in nominal terms. Equation (A31) gives the following relationship then:

$$(1-\alpha)\eta_t \frac{1}{KSN_t} = [\gamma_{U1} + \gamma_{U2}(ucap_t - \overline{ucap})]ucap_t \tag{M13}$$

Differentiating with respect to the price we obtain the first order condition for the price markup (the Lagrange multiplier):

$$\eta_t^j - \frac{\sigma^d - 1}{\sigma^d} + \gamma_P\left(\frac{1}{1+r_t}\pi_{t+1}^j - \pi_t^j\right) = 0 \tag{A32}$$

where $\pi_t^j = P_t^j/P_{t-1}^j - 1$. Using the assumption of symmetry and introducing $\tau = 1/\sigma^d$ equation (A32) is modified as follows. First, we assume that a share sfp of firms determine their prices according to equation (A32), in a forward looking way, while the other $(1 - sfp)$ share of firms are indexing their prices according to inflation. Second, in place of inflation itself, we take the deviation of inflation from its steady state value ($\overline{\pi}$ – inflation target) into account. Third, the markup is augmented by an exogenous shock (u_t^η), and fourth, we use the discount factor (which is a parameter) instead of real interest rate.

$$\eta_t = 1 - \left(\tau + u_t^\eta\right) - \gamma_P[\beta(sfp \cdot \pi_{t+1} + (1 - sfp)\pi_t - \overline{\pi}) - (\pi_t - \overline{\pi})] \tag{M14}$$

The behavior of the final goods producer sector is finally described by the production function, which, at the aggregate level, is given in growth rates on the basis of equation (A23):

$$GY_t = \alpha GA_t + \alpha GL_t\left(1 + \overline{lol}\right) + (1-\alpha)(GK_t + Gucap_t) + (1-\alpha_G)GKG_t \tag{M15}$$

where GY_t, GA_t, GL_t, GK_t, $Gucap_t$ and GKG_t are the growth rates of GDP, labor productivity, labor utilization, capital stock, capacity utilization and public capital stock respectively, whereas $\overline{lol}$ is the steady state value of overhead labor (lol_t).

The intermediate goods sector

Intermediate (or investment) goods are produced by a perfectly competitive sector, using domestic and imported final goods as inputs. The production technology is:

$$I_t = A_t^I I_t^{inp} \tag{A33}$$

where A_t^I is the productivity of the sector, I_t^{inp} is the amount of inputs, being a CES aggregate of domestic and imported final goods with σ^M elasticity of substitution (domestic goods are also CES aggregate of goods, with σ^d elasticity of substitution). The price level of investment goods follows simply:

$$P_t^I = \frac{P_t^C}{A_t^I} \tag{A34}$$

where P_t^C is the price level of final (consumption) goods. The nominal investment to GDP share is determined by the investment to capital stock ratio and the capital stock to GDP ratio. In equation (M16) this relationship is adjusted with the deviation of capital growth rate (GK_t) from its steady state level (see equation (M42)):

$$\ln(ISN_t) = -\ln\left(\frac{1}{KSN_t}\right) + \ln(IK_t) + \overline{GY} + g^{AI} - GK_t \tag{M16}$$

Investments are determined implicitly by the following relationship on the basis of bringing the marginal productivity and the marginal cost of physical capital to parity:

$$\eta_t(1 - t^P)(1 - \alpha)\frac{1}{KSN_t} = Q_t - \left(1 - r_t - \delta - rp - u_t^{rp} - g_{t+1}^{AI} + g^{AI}\right)Q_{t+1} + [\gamma_{U1}(ucap_t - \overline{ucap}) + \gamma_{U2}(ucap_t - \overline{ucap})^2] \tag{M17}$$

3.3.1.3 Labor market and wages

In the model the labor market is also monopolistically competitive. As a consequence, the L_t^j labor demand of firms is a CES aggregate of different types of labor.

$$L_t^j = \left[\int_0^1 L_t^{i,j\frac{\theta-1}{\theta}}\, di\right]^{\frac{\theta}{\theta-1}} \tag{A35}$$

Wage setting is carried out by a union, maximizing the weighted average of the utility of the two household types (we assume that labor types are evenly distributed in the whole population). Reservation wage is given by the standard utility maximizing criteria: real wage (on the basis of consumption price level) equals the ratio of the marginal utilities of leisure and consumption (marginal rate of substitution). When determining reservation wage, the value given by optimization is smoothed by a parameter $wrlag$. Taking consumption and wage taxes into account as well as social security contributions, we have the following formula for real (reservation) wages:

$$\frac{W_t}{P_t^C} = \left(\frac{W_{t-1}}{P_{t-1}^C}\right)^{wrlag} \left[\frac{1}{\eta_t^W}\frac{1+t^C}{1-t_t^W-ssc}\frac{(1-slc)NUL_t^{NR}+slcNUL_t^R}{(1-slc)NUC_t^{NR}+slcNUC_t^R}\right]^{1-wrlag} \tag{A36}$$

where η_t^W is the wage markup. Wage markup evolves according to an equation analogous to the price markup of consumption goods, where a fraction $(1 - sfw)$ of households do not decide on their wage in a forward looking manner but index it to past inflation:

$$\eta_t^W = \frac{\theta-1}{\theta} - \frac{\gamma_W}{\theta}[\beta(\pi_{t+1}^W - (1-sfw)\pi_t) - (\pi_{t+1}^W - (1-sfw)\pi_t)] \quad \text{(A37)}$$

where π_t^W is wage inflation and γ_W is a parameter of the adjustment cost function with respect to wages. We take the combined version of equations (A36) and (A37) into the technical model equations, converted to GDP-shares:

$$(1+t^C)\left[\frac{(1-slc)NUL_t^{NR}+slcNUL_t^R}{(1-slc)NUC_t^{NR}+slcNUC_t^R}\right]^{1-wrlag}\left[\frac{1-t_t^W-ssc}{1+t^C}\frac{\theta-1}{\theta}\frac{1}{YWR_t}\frac{1}{1+GY_t-\overline{GY}}\right]^{wrlag} = \frac{\theta-1}{\theta}\frac{1}{YWR_t}(1-t_t^W-ssc) + \frac{\gamma_W}{\theta}\frac{1}{YWR_t}[(\pi_t^W-\bar{\pi}-\overline{GY})-(1-sfw)(\pi_{t-1}-\bar{\pi})] - \beta\frac{\gamma_W}{\theta}\frac{1}{YWR_t}[(\pi_{t+1}^W-\bar{\pi}-\overline{GY})-(1-sfw)(\pi_t-\bar{\pi})] \quad \text{(M18)}$$

3.3.1.4 Government

The role of the government is modelled by a standard monetary policy reaction function and a sophisticated fiscal block, which operates with fiscal reaction functions similar to the monetary policy rule.

Monetary policy

Monetary policy in the model is described by a Taylor rule:

$$i_t = \tau_{lag}^i i_{t-1} + (1-\tau_{lag}^i)[\bar{r} + \pi_t^T + \tau_\pi^i(\pi_t^C - \pi_t^T) + \tau_{Y1}^i \ln(YGAP_{t-1})] + \tau_{Y2}^i[\ln(YGAP_t) - \ln(YGAP_{t-1})] + u_t^M \quad \text{(M19)}$$

where τ_{lag}^i is a smoothing parameter, τ_π^i, τ_{Y1}^i and τ_{Y2}^i is the reaction parameters of interest rate to the inflation's deviation from its target, the output gap and the change in the output gap, respectively. $YGAP_t$ is a proxy for the output gap (see later), $\bar{r} = 1/(\beta - 1)$ is the natural (steady state) real ineterst rate, π_t^T is the inflation target and u_t^M is an exogenous shock from the side of monetary policy.

Fiscal policy

Fiscal policy is described by similar reaction functions as monetary policy. Fiscal policy operates with five elements on the revenue side: (i) wage income tax, (ii) consumption tax, (ii) capital income tax, (iv) lump sum tax and (v) social security contributions. On the expenditure side we distinguish between (i) transfers, (ii) government consumption and (iii) government investment.

In the case of government consumption, we give a relationship for the change in these expenditures. Government consumption grows in the steady state with the same rate as GDP. Through the output gap we build a counter-cyclical element into the reaction function, and we use the deviation of government consumption from its steady state level among the reaction variables. Finally, we define an exogenous shock and a smoothing behavior. As a result, the following reaction function is written for government consumption:

$$GG_t - \overline{GY} = \tau_{lag}^{CG}(GG_{t-1} - \overline{GY}) + \tau_{adj}^{CG}[\ln(GSN_{t-1}) - \ln(\overline{GSN})] + \tau_0^{CG}[\ln(YGAP_t) - \ln(YGAP_{t-1})] + u_t^{CG} \quad \text{(M20)}$$

where GG_t is the growth rate of government consumption, $\overline{GY}$ is the steady state growth rate of GDP, GSN_t is the nominal share of government consumption in GDP, τ_{lag}^{CG}, τ_{adj}^{CG} and τ_0^{CG} are reaction parameters and u_t^{CG} is the exogenous shock.

We define an analogous reaction function for government investment as in (M20):

$$GIG_t - \overline{GY} - g^{AI} = \tau_{lag}^{IG}(GIG_{t-1} - \overline{GY} - g^{AI}) + \tau_{adj}^{IG}[\ln(IGSN_{t-1}) - \ln(\overline{IGSN})] + \tau_0^{IG}[\ln(YGAP_t) - \ln(YGAP_{t-1})] + u_t^{IG} \quad \text{(M21)}$$

where we use the fact that the steady state growth rate of investments is the sum of the steady state growth rate of GDP and that of the productivity of the intermediate sector.

Transfers are linked to employment counter-cyclically. Define $TRW_t = TR_t/W_t$ as the ratio of per employee nominal transfers to nominal wage. The transfer rule is:

$$TRW_t = \overline{TRW} + \tau^{TR}[1 - L_t - (1 - \bar{L})] + u_t^{TR} \quad \text{(M22)}$$

where $\overline{TRW}$ is the steady state value of transfers, $\bar{L}$ is the steady state employment, τ^{TR} is a reaction parameters and u_t^{TR} is an exogenous shock.

On the revenue side the rate of social security contributions, the capital income tax and the consumption tax is given (ssc, t^P and t^C respectively), we do not define fiscal rules for these revenue elements. The rate of the labor income tax evolves according to

$$t_t^W = \tau_0^W[1 + \tau_1^W \ln(YGAP_t)] \quad \text{(M23)}$$

where τ_0^W is the steady state value of the rate of labor income tax and τ_1^W is a reaction parameter. The role of the lump sum tax is to control the public debt, therefore we define the following rule for it:

$$T_t^{LS} - T_{t-1}^{LS} = \tau_1^{LS}(B_t - \bar{B}) + \tau_2^{LS}(B_t - B_{t-1}) \quad \text{(M24)}$$

where $\bar{B}$ is the target level of the public debt to GDP ratio, τ_1^{LS} and τ_2^{LS} are reaction parameters. The fiscal block is closed by the budget constraint of the government which at the same time defines the dynamics of the public debt:

$$B_t = (1 + r_t - GY_t - g^{pop})B_{t-1} + GSN_t + IGSN_t + \frac{TRW_t}{YWR_t}L_t - (t_t^W + ssc)WS_t - t^P(1 - WS_t) - t^C CSN_t - T_t^{TS} - \varepsilon_t^{GB} \quad \text{(M25)}$$

where WS_t is the nominal wage share in GDP as in equation (M9) and we take into account that GY_t is the growth rate of the GDP per capita. The exogenous disturbance term ε_t^{GB} has a technical role. This variable is not included in the original model estimated for the Eurozone. Its role here is to be able to compensate for the policy interventions appearing on the expenditure side of the government budget on the revenue side. If we were not controlling for this, policy shocks financed by external sources (EU) would lead to spillover effects through increasing deficits and public debt which would bias our results.

Output gap

The output gap is an important variable in the fiscal reaction functions. The model provides an indirect way to measure the output gap. Define the equilibrium employment and capacity utilization as follows:

$$\ln(L_t^{ss}) = \rho^{Lss}\ln(L_{t-1}^{ss}) + (1-\rho^{Lss})L_t \tag{M26}$$

$$ucap_t^{ss} = \rho^{ucap}ucap_{t-1}^{ss} + (1-\rho^{ucap})ucap_t \tag{M27}$$

These two equations give a moving average representation of what is meant to be the potential employment and capacity utilization. According to the production function (A23) we get the following approximate version for the output gap:

$$\ln(YGAP_t) = (1-\alpha)[\ln(ucap_t) - \ln(ucap_t^{ss})] + \alpha[\ln(L_t) - \ln(L_t^{ss})] \tag{M28}$$

3.3.1.5 The foreign sector

The foreign sector appears in two modules. First, we define equations describing the relationship between domestic and foreign variables and second, we model the joint evolution of the variables describing the rest of the world as a mini-model, which drive exogenously the dynamics of the domestic variables.

As it was introduced previously, domestic final absorption (consumption and investment of households and the government) is a CES aggregate of domestic and foreign final goods where the elasticity of substitution between domestic and foreign goods is σ^M. On the basis of this, the demand for import is determined by a parameter describing the (steady state) import share of domestic absorption together with the relative price of imported and domestic goods. The import demand function deriving from this formula is modified by a smoothing parameter in the effect of the relative price. The import demand thus looks like as follows (in nominal terms, expressed relative to the GDP):

$$IMSN_t = (1-s^M)\left[\left(\frac{P_{t-1}^C}{P_{t-1}^M}\right)^{\rho^M}\left(\frac{P_t^C}{P_t^M}\right)^{1-\rho^M}\right]^{\sigma^M}\frac{P_t^M}{P_t^C}(CSN_t + ISN_t + GSN_t + IGSN_t) \tag{M29}$$

where s^M is the share of domestic absorption and ρ^M is the weight of smoothing in the relative price.

We use an analogous expression for exports, using that in the preferences of the foreign sector the elasticity of substitution between domestic and foreign goods is σ^X:

$$EXSN_t = (1-s^M)\left[\left(\frac{(E_{t-1})^{\alpha^X}s^M P_{t-1}}{P_{t-1}^X}\right)^{\rho^X}\left(\frac{(E_t)^{\alpha^X}s^M P_t}{P_t^X}\right)^{1-\rho^X}\right]^{\sigma^X}\frac{P_t^X}{P_t}(YWY_t)^{\alpha^X} \tag{M30}$$

where YWY_t is the ratio of foreign GDP to domestic GDP and α^X is the weight of this ratio in the demand for export.

We apply markup in the price of both the imported and exported goods, for which the same expression is used as introduced for domestic final goods (see equations (M17 and (A34)). The equation for the export markup is:

$$\frac{P_t^X}{P_t} = 1 + u_t^{PX} + \gamma_{PX}[\beta \cdot sfp^X \pi_{t+1}^X + (1 - sfp^X)\pi_{t-1}^X - \bar{\pi}] - (\pi_t^X - \bar{\pi}) \quad \text{(M31)}$$

where γ_{PX} is the usual adjustment parameter, sfp^X is the share of exporters who set prices in a forward looking way, π_t^X is the inflation of export-prices and u_t^{PX} is an exogenous shock.

Similarily for the imported goods:

$$\frac{P_t^M}{P_t} = (E_t)^{\alpha^X}(1 + u_t^{PM} + \gamma_{PM}[\beta \cdot sfp^M \pi_{t+1}^M + (1 - sfp^M)\pi_{t-1}^M - \bar{\pi}] - (\pi_t^M - \bar{\pi})) \quad \text{(M32)}$$

The price level of the consumption goods is thus the weighted average of domestic and imported final goods:

$$\frac{P_t^C}{P_t} = \left[s^M + (1 - s^M)\left(\frac{P_t^M}{P_t}\right)^{1-\sigma^M}\right]^{\frac{1}{1-\sigma^M}} \quad \text{(M33)}$$

The current account is given by the following formula, using exports and imports:

$$NTBSN_t = EXSN_t - IMSN_t + u_t^{EX} \quad \text{(M34)}$$

where u_t^{EX} is an exogenous shock to the current account.

The following equation gives the dynamics of foreign bonds (measured in the domestic currency):

$$B_t^F = (1 + i_t - \pi_{t+1} - GY_t - g^{pop})B_{t-1}^F + NTBSN_t \quad \text{(M35)}$$

where B_t^F is the ratio of the stock of foreign bonds to the domestic GDP.

The relationship between domestic and foreign variables is further specified by the uncovered interest rate parity in (M5) and the purchasing power parity as follows:

$$GE_t + \pi_t^F - \pi_t = \ln\left(\frac{E_t}{E_{t-1}}\right) \quad \text{(M36)}$$

where GE_t is the change in the nominal exchange rate and π_t^F is the foreign inflation.

The mini modal describing the dynamics of the foreign sector contains the deviation of foreign interest rate from its steady state level: $\widehat{\iota^F}_t = i_t^F - \overline{\iota^F}$, the deviation of foreign inflation from its steady state level: $\widehat{\pi^F}_t = \pi_t^F - \overline{\pi^F}$, and the deviation of foreign GDP growth from its steady state level: $\widehat{GYW}_t = GYW_t - \overline{GYW}$, where steady state levels are the para,eters of the model. We define the following VAR(1) model for these three variables:

$$\begin{bmatrix} \widehat{\iota^F}_t \\ \widehat{\pi^F}_t \\ \widehat{GYW}_t \end{bmatrix} = \begin{bmatrix} \rho^{i^F} & \rho^{i^F,\pi^F} & \rho^{i^F,GY^F} \\ \rho^{\pi^F,i^F} & \rho^{\pi^F} & \rho^{\pi^F,GY^F} \\ \rho^{GY^F,i^F} & \rho^{GY^F,\pi^F} & \rho^{GY^F} \end{bmatrix} \begin{bmatrix} \widehat{\iota^F}_{t-1} \\ \widehat{\pi^F}_{t-1} \\ \widehat{GYW}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{iW} \\ \varepsilon_t^{PW} \\ \varepsilon_t^{YW} \end{bmatrix} \quad \text{(M37)-(M39)}$$

3.3.1.6 Balancing equations and identities

The equations introduced so far are closed by several balance identities – these are enumerated in the following.

The GDP identity (final goods market equilibrium) is defined in nominal terms and in GDP shares:

$$1 = CSN_t + ISN_t + IGSN_t + GSN_t + NTBSN_t \quad \text{(M40)}$$

The real interest rate:

$$r_t = i_t - \pi_{t+1} \quad \text{(M41)}$$

The following two equations give the dynamics of private and public capital (their growth rates) respectively:

$$GK_t - (\overline{GY} + g^{AI}) = IK_t - (\delta + g^{pop} + \overline{GY} + g^{AI}) + \varepsilon_t^{CAP} \quad \text{(M42)}$$

$$GKG_t - (\overline{GY} + g^{AI}) = IKG_t - (\delta_G + g^{pop} + \overline{GY} + g^{AI}) + \varepsilon_t^{GCAP} \quad \text{(M43)}$$

In equation (M43) GKG_t stands for the growth rate of the per capita public capital stock, IKG_t is the ratio of government investment to public capital and δ_G is the depreciation rate of public capital. The two exogenous shock variables, ε_t^{CAP} and ε_t^{GCAP} is not defined in the original version of the model specified for the Eurozone. Their role is to have a point where we can implement private investment subsidies' and public infrastructure spending's effect on the respective capital stocks.

The definition of the above capital growth rates (in a combined way):

$$GIG_t - GI_t = \ln(IGSN_t) - \ln(ISN_t) - \ln(IGSN_{t-1}) + \ln(ISN_{t-1}) \quad \text{(M44)}$$

The identities describing the relationship between investment and capital stock in the two sectors:

$$GI_t - GK_{t-1} = \ln(IK_t) - \ln(IK_{t-1}) \quad \text{(M45)}$$

$$GIG_t - GKG_{t-1} = \ln(IKG_t) - \ln(IKG_{t-1}) \quad \text{(M46)}$$

The growth rate of the private capital stock:

$$GY_t - GK_t + g_t^{AI} = \ln\left(\frac{1}{KSN_t}\right) - \ln\left(\frac{1}{KSN_{t-1}}\right) \quad \text{(M47)}$$

The definition of disposable income:

$$WSW_t = (1 - t_t^W - ssc)WS_t \tag{M48}$$

The money stock to GDP ratio in function of the interest rate:

$$MRY_t = (1 + i_t)^{\varphi} \tag{M49}$$

The growth rate of consumption, for total consumption, consumption of ricardian and non ricardian households respectively:

$$GC_t - GY_t + \pi_t^C - \pi_t = \ln(CSN_t) - \ln(CSN_{t-1}) \tag{M50}$$

$$GC_t^R - GY_t + \pi_t^C - \pi_t = \ln(CSN_t^R) - \ln(CSN_{t-1}^R) \tag{M51}$$

$$GC_t^{NR} - GY_t + \pi_t^C - \pi_t = \ln(CSN_t^{NR}) - \ln(CSN_{t-1}^{NR}) \tag{M52}$$

Similarly, the growth rate of exports, imports and government consumption:

$$GEX_t - GY_t + \pi_t^X - \pi_t = \ln(EXSN_t) - \ln(EXSN_{t-1}) \tag{M53}$$

$$GIM_t - GY_t + \pi_t^M - \pi_t = \ln(IMSN_t) - \ln(IMSN_{t-1}) \tag{M54}$$

$$GG_t - GY_t + \pi_t^C - \pi_t = \ln(GSN_t) - \ln(GSN_{t-1}) \tag{M55}$$

The growth rate of employment:

$$GL_t = \ln(L_t) - \ln(L_{t-1}) \tag{M56}$$

The (nominal) ratio of transfers to GDP:

$$TRSN_t = TRW_t \frac{L_t}{YWR_t} \tag{M57}$$

Net transfers:

$$NTRSN_t = TRW_t \frac{L_t}{YWR_t} - T_t^{LS} \tag{M58}$$

The growth rate of lump sum tax:

$$GTAX_t - GY_t - \pi_t = \ln(T_t^{LS}) - \ln(T_{t-1}^{LS}) \tag{M59}$$

The growth rate of transfers:

$$GTR_t - GL_t - \pi_t^{WR} = \ln(TRW_t) - \ln(TRW_{t-1}) \tag{M60}$$

The growth rate of capacity utilization:

$$Gucap_t = \ln(ucap_t) - \ln(ucap_{t-1}) \tag{M61}$$

The growth rate of TFP adjusted by capacity utilization:

$$GAU_t = (1 - \alpha) \cdot Gucap_t + \alpha \cdot GA_t \tag{M62}$$

The growth rate of the ratio of rela wage to GDP:

$$GWRY_t = \ln(YWR_t) - \ln(YWR_{t-1}) \tag{M63}$$

The growth rate of the foreign GDP:

$$\ln(YWY_t) - \ln(YWY_{t-1}) = GYW_t - GY_t \tag{M64}$$

The change in the output gap:

$$GY_t - GPOT_t = \ln(YGAP_t) - \ln(YGAP_{t-1}) \tag{M65}$$

The change in public debt:

$$DB_t = B_t - B_{t-1} \tag{M66}$$

Identities with the price levels and inflations of consumption goods, imports and exports:

$$\pi_t^C - \pi_t = \ln\left(\frac{P_t^C}{P_t}\right) - \ln\left(\frac{P_{t-1}^C}{P_{t-1}}\right) \tag{M67}$$

$$\pi_t^M - \pi_t = \ln\left(\frac{P_t^M}{P_t}\right) - \ln\left(\frac{P_{t-1}^M}{P_{t-1}}\right) \tag{M68}$$

$$\pi_t^X - \pi_t = \ln\left(\frac{P_t^X}{P_t}\right) - \ln\left(\frac{P_{t-1}^X}{P_{t-1}}\right) \tag{M69}$$

The growth rate of nominal wages:

$$-\pi_t^W + GY_t + \pi_t = \ln(YWR_t) - \ln(YWR_{t-1}) \tag{M70}$$

The growth rate of real wages:

$$\pi_t^W = \pi_t^{WR} + \pi_t \tag{M71}$$

As the model is written in terms of per capita variables, the following equations give the level growth rates of the main macro variables (GDP, household consumption, investment, government consumption, exports and imports):

$$GY_t^{lev} = GY_t + g^{pop} \tag{M72}$$

$$GC_t^{lev} = GC_t + g^{pop} \tag{M73}$$

$$GI_t^{lev} = GI_t + g^{pop} + \varepsilon_t^{INV} \tag{M74}$$

$$GG_t^{lev} = GG_t + g^{pop} \tag{M75}$$

$$GEX_t^{lev} = GEX_t + g^{pop} + dgex \tag{M76}$$

$$GIM_t^{lev} = GIM_t + g^{pop} + dgim \quad \text{(M77)}$$

The exogenous shock variable ε_t^{INV} in equation (M47) ha a technical character, it is not used in the original specification for the Eurozone. Its role is to implement private investment subsidies into the model.

The change in the absolute level of import and export prices:

$$\pi_t^{M,lev} = \pi_t^M + dgpm \quad \text{(M78)}$$

$$\pi_t^{X,lev} = \pi_t^X + dgpx \quad \text{(M79)}$$

The household and government consumption to GDP ratios in real terms:

$$\ln(CY_t) = \ln(CSN_t) - \ln\left(\frac{P_t^C}{P_t}\right) \quad \text{(M80)}$$

$$\ln(GGY_t) = \ln(GSN_t) - \ln\left(\frac{P_t^C}{P_t}\right) \quad \text{(M81)}$$

The equations of the model contain several variables also in logarithm. In the description above all logarithms were rewritten in non-logarithmized form, but to be complete with the technical equations, we present here the identities resulting from these dualities. Equation (M9) in logarithms:

$$\ln(WS_t) = \ln(L_t) - \ln(YWR_t) \quad \text{(M82)}$$

And further:

$$B_t = \exp(\ln(B_t)) \quad \text{(M83)}$$

$$CSN_t^{NR} = \exp(\ln(CSN_t^{NR})) \quad \text{(M84)}$$

$$GSN_t = \exp(\ln(GSN_t)) \quad \text{(M85)}$$

$$TRSN_t = \exp(\ln(TRSN_t)) \quad \text{(M86)}$$

3.3.1.7 Exogenous processes

The model contains several exogenous shock variables which are determined by the following equations (the content of the different exogenous variables were given previously). Parameters ρ measure the respective persistences while the variables ε_t are the white noises driving the exogenous variables with zero mean and a respective standard deviation σ.

$$GA_t = g^A + \varepsilon_t^Y \quad \text{(M87)}$$

$$lol_t - \overline{lol} = \rho^{lol}\left(lol_{t-1} - \overline{lol}\right) + \varepsilon_t^{lol} \quad \text{(M88)}$$

$$g_t^{AI} = g^{AI} + u_t^{AI} \quad \text{(M89)}$$

$$u_t^C = \rho^C u_{t-1}^C + \varepsilon_t^C \quad \text{(M90)}$$

$$u_t^{\eta} = \rho^{\eta} u_{t-1}^{\eta} + \varepsilon_t^{\eta} \quad \text{(M91)}$$

$$u_t^{PM} = \rho^{PM} u_{t-1}^{PM} + \varepsilon_t^{PM} \quad \text{(M92)}$$

$$u_t^{PX} = \rho^{PX} u_{t-1}^{PX} + \varepsilon_t^{PX} \quad \text{(M93)}$$

$$u_t^{EX} = \rho^{EX} u_{t-1}^{EX} + \varepsilon_t^{EX} \quad \text{(M94)}$$

$$u_t^{CG} = \rho^{CG} u_{t-1}^{CG} + \varepsilon_t^{CG} \quad \text{(M95)}$$

$$u_t^{IG} = \rho^{IG} u_{t-1}^{IG} + \varepsilon_t^{IG} \quad \text{(M96)}$$

$$u_t^{L} = \rho^{L} u_{t-1}^{L} + \varepsilon_t^{L} \quad \text{(M97)}$$

$$u_t^{M} = \varepsilon_t^{M} \quad \text{(M98)}$$

$$u_t^{AI} = \rho_1^{AI} u_{t-1}^{AI} + \rho_1^{AI} u_{t-2}^{AI} + \rho_1^{AI} u_{t-3}^{AI} + \rho_1^{AI} u_{t-4}^{AI} + \varepsilon_t^{AI} \quad \text{(M99)}$$

$$u_t^{F} = \rho^{F} u_{t-1}^{F} + \varepsilon_t^{F} \quad \text{(M100)}$$

$$u_t^{rp} = \rho^{rp} u_{t-1}^{rp} + \varepsilon_t^{rp} \quad \text{(M101)}$$

$$u_t^{W} = \varepsilon_t^{W} \quad \text{(M102)}$$

$$u_t^{TR} = \rho^{TR} u_{t-1}^{TR} + \varepsilon_t^{TR} \quad \text{(M103)}$$

$$\pi_t^{T} - \bar{\pi} = 0 \quad \text{(M104)}$$

3.3.1.8 Variables and parameters of the model

The endogenous variables of the model are summarized by Table 7. This table lists the technical variables of the model and the normal and logarithmized forms are denoted according to this.

Table 7 – Endogenous variables of the MACRO model

#	Notation	Definition
1.	$\ln(NUC_t^R)$	The ratio of marginal utility of consumption to the GDP for the ricardian households (nominal)
2.	NUL_t^R	The ratio of marginal utility of consumption to the GDP for the ricardian households (nominal)
3.	$\ln(NUC_t^{NR})$	The ratio of marginal utility of consumption to the GDP for the liquidity constrained households (nominal)
4.	NUL_t^{NR}	The ratio of marginal utility of consumption to the GDP for the liquidity constrained households (nominal)
5.	CSN_t^{NR}	The consumption to GDP ratio of liquidity constrained households (nominal)
6.	$\ln(CSN_t^R)$	The consumption to GDP ratio of ricardian households (nominal)
7.	$\ln(CSN_t^{NR})$	The consumption to GDP ratio of liquidity constrained households (nominal)
8.	$\ln(CSN_t)$	Consumption to GDP ratio (nominal)
9.	$\ln(ISN_t)$	Investment to GDP ratio (nominal)
10.	$\ln(GSN_t)$	Government consumption to GDP ratio (nominal)
11.	GSN_t	Government consumption to GDP ratio (nominal)
12.	$\ln(IGSN_t)$	Government investment to GDP ratio (nominal)
13.	$\ln(EXSN_t)$	Export to GDP ratio (nominal)

14.	$\ln(IMSN_t)$	Import to GDP ratio (nominal)
15.	$NTBSN_t$	Net export to GDP ratio (nominal)
16.	T_t^{LS}	Lump sum tax to GDP ratio (nominal)
17.	$TRSN_t$	Transfers to GDP ratio (nominal)
18.	$\ln(TRSN_t)$	Transfers to GDP ratio (nominal)
19.	$NTRSN_t$	Net transfers (by lump sum taxes) to GDP ratio (nominal)
20.	$BGYN_t$	Public debt to GDP ratio (nominal)
21.	$\ln(BGYN_t)$	Public debt to GDP ratio (nominal)
22.	$\ln(KSN_t)$	Private capital stock to GDP ratio (nominal)
23.	$\ln(YWY_t)$	Foreign GDP to domestic GDP ratio (nominal)
24.	$\ln(YWR_t)$	The ratio of GDP to nominal wages
25.	WS_t	Wages to GDP ratio (nominal)
26.	$\ln(WS_t)$	Wages to GDP ratio (nominal)
27.	WSW_t	Disposable income to GDP ratio (nominal)
28.	$\ln(MRY_t)$	Money stock to GDP ratio (nominal)
29.	DB_t	Government deficit to GDP ratio (nominal)
30.	B_t^F	External debt to GDP ratio (nominal)
31.	$\ln(E_t)$	Exchange rate (nominal)
32.	i_t	Interest rate (nominal)
33.	i_t^F	Foreign interest rate (nominal)
34.	r_t	Real interest rate
35.	Q_t	Tobin Q
36.	$\ln(L_t)$	Employment rate
37.	L_t^{ss}	Equilibrium employment rate (moving average)
38.	lol_t	The share of overhead labor in employment
39.	$ucap_t$	Capacity utilization
40.	$ucap_t^{ss}$	Equilibrium capacity utilization (moving average)
41.	TRW_t	The ratio of per employee transfers to real wage
42.	η_t	The inverse of markup factor in the final goods sector
43.	GC_t^R	The growth rate of consumption of ricardian households
44.	GC_t^{NR}	The growth rate of consumption of liquidity constrained households
45.	GC_t	The growth rate of per capita consumption
46.	GC_t^{lev}	The growth rate of consumption
47.	GI_t	The growth rate of per capita investment
48.	GI_t^{lev}	The growth rate of investment
49.	GG_t	The growth rate of per capita government consumption
50.	GG_t^{lev}	The growth rate of government consumption
51.	GIG_t	The growth rate of per capita government investment
52.	GEX_t	The growth rate of per capita exports
53.	GEX_t^{lev}	The growth rate of exports
54.	GIM_t	The growth rate of per capita imports
55.	GIM_t^{lev}	The growth rate of imports
56.	GE_t	The growth rate of the exchange rate
57.	GK_t	The growth rate of private capital stock
58.	GKG_t	The growth rate of public capital stock
59.	GL_t	The growth rate of employment rate
60.	$GTAX_t$	The growth rate of lump sum tax
61.	GA_t	The growth rate of TFP
62.	GAU_t	The growth rate of TFP adjusted by capacity utilization
63.	g_t^{AI}	The growth rate of the productivity of intermediate goods
64.	GTR_t	The growth rate of transfers
65.	GUC_t	The growth rate of the marginal utility of consumption (ricardian households)

66.	$Gucap_t$	The growth rate of capacity utilization
67.	$GWRY_t$	The growth rate of the ratio of rela wage to GDP
68.	GY_t	The growth rate of per capita GDP
69.	GY_t^{lev}	The growth rate of GDP
70.	$GPOT_t$	The change in potential GDP (proxy)
71.	GYW_t	The growth rate of foreign GDP
72.	$\ln(IK_t)$	The ratio if investment to capital stock in the private sector
73.	$\ln(IKG_t)$	The ratio if investment to capital stock in the public sector
74.	$\ln(YGAP_t)$	Output gap
75.	$\ln(P_t^C/P_t)$	The relative price of consumption goods
76.	$\ln(P_t^M/P_t)$	The relative price of import
77.	$\ln(P_t^X/P_t)$	The relative price of export
78.	π_t	Domestic inflation
79.	π_t^F	Foreign inflation
80.	π_t^C	Inflation of consumption goods
81.	π_t^M	Inflation of import goods
82.	π_t^X	Inflation of export goods
83.	$\pi_t^{M,lev}$	Inflation of import goods with trend
84.	$\pi_t^{X,lev}$	Inflation of export goods with trend
85.	π_t^W	Wage inflation
86.	π_t^{WR}	The growth rate of real wages
87.	t_t^W	The tax rate for income tax
88.	π_t^T	Inflation target
89.	$\ln(CY_t)$	Consumption to GDP ratio (real)
90.	$\ln(GGY_t)$	Government consumption to GDP ratio (real)
91.	u_t^C	Shock to consumption preference
92.	u_t^η	Shock to markup
93.	u_t^{PX}	Shock to export prices
94.	u_t^{PM}	Shock to import prices
95.	u_t^{EX}	Shock to current account
96.	u_t^{CG}	Shock to government consumption
97.	u_t^{IG}	Shock to government investment
98.	u_t^L	Shock to leisure preference
99.	u_t^M	Shock to monetary policy
100.	u_t^{AI}	Shock to the productivity of the intermediate goods sector
101.	u_t^F	Shock to foreign risk premium
102.	u_t^{rp}	Shock to risk premium on physical cpital
103.	u_t^{TR}	Shock to transfers
104.	u_t^W	Shock to labor demand

The exogenous variables are summarized in Table 8.

Table 8 – The exogenous variables of the MACRO model

#	Notation	Definition
1.	ε_t^C	Shock to consumption preference
2.	ε_t^η	Shock to markup
3.	ε_t^{PX}	Shock to export prices
4.	ε_t^{PM}	Shock to import prices
5.	ε_t^{EX}	Shock to current account
6.	ε_t^{CG}	Shock to government consumption

7.	ε_t^{IG}	Shock to government investment
8.	ε_t^{L}	Shock to leisure preference
9.	ε_t^{M}	Shock to monetary policy
10.	ε_t^{AI}	Shock to the productivity of the intermediate goods sector
11.	ε_t^{F}	Shock to foreign risk premium
12.	ε_t^{rp}	Shock to risk premium on physical cpital
13.	ε_t^{TR}	Shock to transfers
14.	ε_t^{W}	Shock to labor demand
15.	ε_t^{lol}	Shock to overhead labor
16.	ε_t^{PW}	Shock to foreign inflation
17.	ε_t^{YW}	Shock to foreign GDP
18.	ε_t^{iW}	Shock to foreign interest rate
19.	ε_t^{Y}	Shock to TFP
20.	ε_t^{INV}	Shock to private investment
21.	ε_t^{CAP}	Shock to private capital stock growth
22.	ε_t^{CAP}	Shock to public capital stock growth
23.	ε_t^{GB}	Shock to government budget revenues

The parameters of the model are summarized in Table 9.

Table 9 – The parameters of the MACRO model

#	Notation	Definition
1.	γ_{U1}	Cost parameter of capacity utilization 1
2.	γ_{U2}	Cost parameter of capacity utilization 2
3.	α^X	The elasticity of exports to foreign GDP
4.	α	The production elasticity of labor
5.	α^G	The additive inverse of the production elasticity of public capital
6.	β	Discount factor
7.	τ_1^{LS}	The reaction of lump sum tax on its deviation from target
8.	τ_2^{LS}	The reaction of lump sum tax on change in public debt
9.	$\bar{B}$	The public debt to GDP target
10.	δ	Depreciation rate for the private capital
11.	δ^G	Depreciation rate for the public capital
12.	$dgex$	The empirical trend of the export to GDP ratio
13.	$dgim$	The empirical trend of the import to GDP ratio
14.	$dgpm$	The empirical trend of the import price level
15.	$dgpx$	The empirical trend of the export price level
16.	$\bar{\iota}^F$	The steady state foreign interest rate
17.	τ_0^{CG}	The reaction of government consumption (growth) on past change in the output gap
18.	γ_I	Adjustment cost parameter of physical capital investments
19.	γ_K	Adjustment cost parameter of physical capital investments
20.	γ_L	Parameter of the adjustment cost function for labor
21.	γ_P	Parameter of the adjustment cost function for price
22.	γ_{PM}	The weight of inflation indexing in the import markup
23.	γ_{PX}	The weight of inflation indexing in the export markup
24.	γ_W	Parameter of the adjustment cost function for wage
25.	$\bar{\pi}$	Inflation target
26.	g^{AI}	The steady state growth rate of the productivity of the intermediate sector
27.	g^{pop}	Population growth rate
28.	$\overline{\pi^F}$	Foreign inflation target

29.	τ_{lag}^{CG}	The smoothing parameter of government consumption
30.	τ_{adj}^{CG}	The reaction of government consumption (growth) on the deviation of G/Y from steady state
31.	$\overline{GSN}$	The steady state ratio of government consumption to GDP
32.	g^A	The steady state growth rate of TFP
33.	$\overline{GY}$	The steady state growth rate of per capita GDP
34.	$\overline{GYW}$	The steady state growth rate of foreign GDP
35.	h^C	Habit parameter in consumption
36.	h^L	Habit parameter in leisure
37.	τ_{lag}^{IG}	The smoothing parameter of government investment
38.	τ_{adj}^{IG}	The reaction of government investment (growth) on the deviation of GI/Y from steady state
39.	τ_{lag}^{i}	The parameter for interest rate smoothing
40.	τ_0^{IG}	The reaction of government investment (growth) on past change in the output gap
41.	$\overline{IGSN}$	The steady state ratio of government investment to GDP
42.	κ	Parameter of the utility function
43.	$\bar{L}$	The steady state employment rate
44.	$\overline{lol}$	The steady state share of overhead labor
45.	$\overline{YWY}$	The steady state value of the log ratio of foreign and domestic GDP
46.	ω	Parameter of the utility function
47.	ρ^C	Persistence parameter, consumption preference shock
48.	ρ^η	Persistence parameter, markup shock
49.	ρ^{PM}	Persistence parameter, import markup shock
50.	ρ^{PX}	Persistence parameter, export markup shock
51.	ρ^{EX}	Persistence parameter, current account shock
52.	ρ^{CG}	Persistence parameter, government consumption shock
53.	ρ^{IG}	Persistence parameter, government investment shock
54.	ρ^{Lss}	Smoothing parameter in equilibrium employment
55.	ρ^L	Persistence parameter, leisure preference shock
56.	ρ^{lol}	Persistence parameter, overhead labor shock
57.	ρ_1^{AI}	Persistence parameter, intermediate sector productivity shock, lag1
58.	ρ_2^{AI}	Persistence parameter, intermediate sector productivity shock, lag2
59.	ρ_3^{AI}	Persistence parameter, intermediate sector productivity shock, lag3
60.	ρ_4^{AI}	Persistence parameter, intermediate sector productivity shock, lag4
61.	ρ^M	The weight of past prices in import share
62.	ρ^X	The weight of past prices in export share
63.	ρ^F	Persistence parameter, foreign risk premium shock
64.	ρ^{rp}	Persistence parameter, physical investment risk premium shock
65.	ρ^{ucap}	Smoothing parameter in equilibrium capacity utilization
66.	ρ^{i^F}	Smoothing parameter of foreign interest rate
67.	ρ^{i^F,π^F}	Effect of foreign inflation on foreign interest rate
68.	ρ^{i^F,GY^F}	Effect of foreign GDP on foreign interest rate
69.	ρ^{π^F,i^F}	Effect of foreign interest rate on foreign inflation
70.	ρ^{π^F}	Smoothing parameter of foreign inflation
71.	ρ^{π^F,GY^F}	Effect of foreign GDP on foreign inflation
72.	ρ^{GY^F,i^F}	Effect of foreign interest rate on foreign GDP
73.	ρ^{GY^F,π^F}	Effect of foreign inflation on foreign GDP
74.	ρ^{GY^F}	Smoothing parameter of foreign GDP
75.	$\rho^{GY^F,GY}$	Effect of the rate of domestic to foreign GDP foreign inflation on foreign GDP
76.	rf	The effect of external debt on foreign risk premium
77.	rp	Risk premium on physical capital
78.	ω^X	The share of domestic consumption

79.	sfp	The share of forward looking firms (final consumption goods)
80.	sfp^M	The share of forward looking firms (import goods)
81.	sfp^X	The share of forward looking firms (export goods)
82.	sfw	The share of forward looking households (wage setting)
83.	σ^C	Parameter of the utility function
84.	σ^X	Foreign elasticity of substitution between domestic and foreign goods
85.	σ	Domestic elasticity of substitution between domestic and foreign goods
86.	slc	The share of liquidity constrained households
87.	ssc	Social security contribution rate
88.	τ	Inverse of the elasticity of substitution between domestic varieties
89.	t^P	Tax rate of capital income
90.	θ	Elasticity of substitution between labor types
91.	τ_π^i	The reaction of the interest rate on inflation (Taylor rule)
92.	τ^{TR}	The effect of employment on transfers
93.	$\overline{TRW}$	The steady state level of transfers (transfer to wage ratio)
94.	ρ^{TR}	Persistence parameter, transfers shock
95.	τ_Y^i	The reaction of the interest rate on output gap (Taylor rule)
96.	t^C	VAT rate
97.	τ_0^W	Steady state rate of labor income tax
98.	τ_1^W	The effect of output gap on labor income tax rate
99.	$\overline{ucap}$	The steady state capacity utilization
100.	$wrlag$	Smoothing parameter in wage setting
101.	φ	The elasticity of money stock to interest rate
102.	σ_ε^{iW}	The standard deviation of the foreign interest rate shock
103.	σ_ε^{PW}	The standard deviation of the foreign inflation shock
104.	σ_ε^{YW}	The standard deviation of the foreign GDP shock
105.	σ_ε^{AI}	The standard deviation of the intermediate sector productivity shock
106.	σ_ε^{GB}	The standard deviation of the budget revenue shock
107.	σ_ε^{INV}	The standard deviation of the private invetsment shock
108.	σ_ε^{C}	The standard deviation of the consumption preference shock
109.	$\sigma_\varepsilon^{\eta}$	The standard deviation of the markup shock
110.	σ_ε^{PM}	The standard deviation of the import price shock
111.	σ_ε^{PX}	The standard deviation of the export price shock
112.	σ_ε^{EX}	The standard deviation of the current account shock
113.	σ_ε^{CG}	The standard deviation of the government consumption shock
114.	σ_ε^{IG}	The standard deviation of the government investment shock
115.	σ_ε^{L}	The standard deviation of the leisure preference shock
116.	σ_ε^{lol}	The standard deviation of the overhead labor shock
117.	σ_ε^{M}	The standard deviation of the monetary policy shock
118.	σ_ε^{F}	The standard deviation of the foreign risk premium shock
119.	σ_ε^{rp}	The standard deviation of the physical capital risk premium shock
120.	σ_ε^{TR}	The standard deviation of the transfers shock
121.	σ_ε^{W}	The standard deviation of the labor demand shock
122.	σ_ε^{Y}	The standard deviation of the TFP shock
123.	σ_ε^{INV}	The standard deviation of the investment growth shock
124.	σ_ε^{CAP}	The standard deviation of the private capital growth shock
125.	$\sigma_\varepsilon^{GCAP}$	The standard deviation of the public capital growth shock
126.	σ_ε^{GB}	The standard deviation of the government revenue shock

3.3.2 Solving the model

The DSGE model defined by equations (M1)-(M104) is solved by standard algorithms used in the literature, with the help of Dynare, a dedicated software for solving and estimating this type of models (see Adjemian et al., 2011). Denote the vector of endogenous variables by $\mathbf{y}$, the vector of exogenous variables by $\boldsymbol{\varepsilon}_t$ and the vector of parameters is $\boldsymbol{\theta}$. The model (M1)-(M104) can be written in compact form as follows, explicitly stating the role of rational expectations:[11]

$$E_t[F(\mathbf{y}_{t-1},\mathbf{y}_t,\mathbf{y}_{t+1},\boldsymbol{\varepsilon}_t,\boldsymbol{\theta})]=\mathbf{0} \tag{A39}$$

where E_t is the expectations operator. The solution of the model is a function

$$\mathbf{y}_t = g(\mathbf{y}_{t-1},\boldsymbol{\varepsilon}_t) \tag{A40}$$

which satisfies the system of equations (A42). Instead of exactly finding the function $g(\cdot)$, the standard solution is to take the first or second order approximation to the model. The generally used method follows the algorithm of Uhlig (1999) which constitutes of the following steps (see for example Horváth, 2006):

1. Write the equations of the model. These consist of the first order conditions following from actors' decisions and conditions for market equilibria. This step is given by the relationships from (M1) to (M104) or in compact form, equation (A39).
2. Calculating the steady state of the model. This means finding a vector $\bar{\mathbf{y}}=\mathbf{y}_{t-1}=\mathbf{y}_t=\mathbf{y}_{t+1}$ of endogenous variables such that it satisfies the system in (A39) given that there are no shocks ($\boldsymbol{\varepsilon}_t=\mathbf{0}$):
   $$F(\bar{\mathbf{y}},\bar{\mathbf{y}},\bar{\mathbf{y}},\boldsymbol{\theta})=\mathbf{0} \tag{A41}$$
   On the basis of this, the steady state can be written in function of the model parameters:
   $$\bar{\mathbf{y}}=s(\boldsymbol{\theta}) \tag{A42}$$
   It is possible to solve for the steady state a given parameter vector using standard methods (e.g. Newton's method). In the case of our model (M1)-(M104), though, the steady state can be given by simple, logical reasoning (as a consequence of the definitions in growth rates and shares). The determination of the steady state is given in detail in the following subsection.
3. Loglinearizing the model equations around the steady state. This can be done by recasting the equations into Taylor series. As a result, the system of equations in (A39) can be written in the following matrix form:
   $$\mathbf{y}_t = E_t[\mathbf{A}(\boldsymbol{\theta})\mathbf{y}_{t+1}]+\mathbf{B}(\boldsymbol{\theta})\mathbf{y}_t+\mathbf{C}(\boldsymbol{\theta})\mathbf{y}_{t-1}+\mathbf{D}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t \tag{A43}$$
4. The solution to (A43) is (using (A40)) is the matrix equation
   $$\mathbf{y}_t=\mathbf{F}(\boldsymbol{\theta})\mathbf{y}_{t-1}+\mathbf{G}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t \tag{A44}$$
   so the exercise is to find the matrices $\mathbf{F}(\boldsymbol{\theta})$ and $\mathbf{G}(\boldsymbol{\theta})$. This can be done by the method of Blanchard-Kahn (1980) or the method of generalized eigenvalues, among others
5. Using the solution in (A44) we can analyze the model and run simulations.

3.3.2.1 The steady state

[11] For the sake of preciseness, it is due to note that the model, in its form defined by (M1)-(M104) contains one period forward and four periods backward looking (see equation (M99)). Using three auxiliary equations, though, the model can be reformulated as in (A38).

In the steady state of the model the endogenous variables are constant which corresponds to a balanced growth path in the case of a decently specified model. The structure of the model gives simple rules for the steady state values of the different endogenous variables. The steady state growth rate of the domestic GDP ($\overline{GY}$), the domestic inflation target ($\bar{\pi}$), the population growth rate (g^{pop}) and the productivity growth of the intermediate sector (g^{AI}) determine the steady state of most of the variables.

The inflation target determines the GDP deflator, and the inflation of consumption goods, intermediate goods, import and export prices:

$$\pi_t = \pi_t^C = \pi_t^I = \pi_t^M = \pi_t^X = \bar{\pi} \tag{A45}$$

The following two equations give the import and export inflations with trend (see equations (M78 and (M79)):

$$\pi_t^{M,lev} = \bar{\pi} + dgpm \tag{A46}$$

$$\pi_t^{X,lev} = \bar{\pi} + dgpx \tag{A47}$$

The steady state growth rate of the per capita GDP and the elements of its expenditure side is given by the steady state growth rate of GDP:

$$GY_t = GC_t = GC_t^R = GC_t^{NR} = GG_t = GEX_t = GIM_t = \overline{GY} \tag{A48}$$

The growth rates of private and public investment is determined by the productivity growth rate of the intermediate sector (see equation (A33)):

$$GI_t = GIG_t = GK_t = GKG_t = \overline{GY} + g^{AI} \tag{A49}$$

In addition to the per capita growth rates, the level growth rates follow logically:

$$GY_t^{lev} = GC_t^{lev} = GG_t^{lev} = GEX_t^{lev} = GIM_t^{lev} = \overline{GY} + g^{pop} \tag{A50}$$

$$GI_t^{lev} = \overline{GY} + g^{AI} + g^{pop} \tag{A51}$$

The respective steady state parameters define the steady state values of the following variables (respectively: employment rate, capacity utilization, government consumption to GDP ratio, government investment to GDP ratio, transfers to wage ratio, public debt to GDP ratio, ratio of foreign and domestic GDP, share of overhead labor):

$$L_t = \bar{L} \tag{A52}$$

$$ucap_t = \overline{ucap} \tag{A53}$$

$$GSN_t = \overline{GSN} \tag{A54}$$

$$IGSN_t = \overline{IGSN} \tag{A55}$$

$$TRW_t = \overline{TRW} \tag{A56}$$

$$B_t = \bar{B} \tag{A57}$$

$$YWY_t = \overline{YWY} \tag{A58}$$

$$lol_t = \overline{lol} \tag{A59}$$

Following from the VAR model written for foreign variables (interest rate, inflation, GDP), the steady state of them is defined by the respective steady state parameters:

$$i_t^F = \overline{\iota^F} \tag{A60}$$

$$\pi_t^F = \overline{\bar{\pi}^F} \tag{A61}$$

$$GYW_t = \overline{GYW} \tag{A62}$$

Following from equations (M26) and (M27):

$$L_t^{ss} = \bar{L} \tag{A63}$$

$$ucap_t^{ss} = \overline{ucap} \tag{A64}$$

Using (M28) and the equations right above:

$$YGAP_t = 1 \tag{A65}$$

The subsequent equations follow from those right above and from equations (M56), (M61), (M60), (M63), (M66), (M65) and (M3) respectively.

$$GL_t = 0 \tag{A66}$$

$$Gucap_t = 0 \tag{A67}$$

$$GTR_t = \overline{GY} \tag{A68}$$

$$GYWR_t = 0 \tag{A69}$$

$$DB_t = 0 \tag{A70}$$

$$GPOT_t = \overline{GY} \tag{A71}$$

$$GUC_t = 0 \tag{A72}$$

The steady state interest rate using the Taylor rule is:

$$i_t = \frac{1-\beta}{\beta} + \bar{\pi} \tag{A73}$$

The steady state for the real interest rate is thus (see equation (M41)):

$$r_t = \frac{1-\beta}{\beta} \tag{A74}$$

Using (M14) we get the following steady state for the markup in the final goods sector:

$$\eta_t = 1 - \tau \tag{A75}$$

Using (M31), (M32) and (M33) the steady states of relative prices are:

$$\frac{P_t^X}{P_t} = 1 \tag{A76}$$

$$\frac{P_t^M}{P_t} = \bar{E}^{\alpha^X} \tag{A77}$$

$$\frac{P_t^C}{P_t} = \left[s^M + (1 - s^M)\left(\bar{E}^{\alpha^X}\right)^{1-\sigma^M} \right]^{\frac{1}{1-\sigma^M}} \tag{A78}$$

where $\bar{E}$ is the steady state exchange rate which is normalized to 1 during the simulations.

The steady state growth rate of TFP ~~follrom~~ the production function (M15):

$$GA_t = \frac{\alpha+\alpha^G-1}{\alpha}\overline{GY} - \frac{2-\alpha-\alpha^G}{\alpha} g^{AI} \tag{A79}$$

The TFP growth adjusted for capacity utilization:

$$GAU_t = \alpha\left(\frac{\alpha+\alpha^G-1}{\alpha}\overline{GY} - \frac{2-\alpha-\alpha^G}{\alpha} g^{AI}\right) \tag{A80}$$

Using (M12), the steady state for the real wage to GDP ratio is

$$YWR_t = \frac{\alpha}{(1-\tau)(1+\overline{lol})\bar{L}} \tag{A81}$$

The steady state for the wage share follows from equation (M9):

$$WS_t = \bar{L}\frac{(1-\tau)(1+\overline{lol})\bar{L}}{\alpha} \tag{A82}$$

It follows from equation (M17) that

$$KSN_t = \frac{(1-\tau)(1-t^P)(1-\alpha)}{(1-\beta)/\beta+\delta+rp} \tag{A83}$$

According to equations (M42) and (M43) the steady state of the ratio of investment to capital stock in the private and public sectors respectively is:

$$IK_t = \delta + g^{AI} + \overline{GY} + g^{pop} \tag{A84}$$

$$IKG_t = \delta^G + g^{AI} + \overline{GY} + g^{pop} \tag{A85}$$

From equation (M16) follows the steady state investment to GDP share:

$$ISN_t = (\delta + g^{AI} + \overline{GY} + g^{pop})\frac{(1-\beta)/\beta+\delta+rp}{(1-\tau)(1-t^P)(1-\alpha)} \tag{A86}$$

The steady state of the external debt stock can be determined using equation (M5):

$$B_t^F = \frac{\overline{\iota^F} - \overline{\pi^F} - (1-\beta)/\beta}{rf} \tag{A87}$$

Equation (M35) determines the share of net exports to GDP:

$$NTBSN_t = \frac{\overline{\iota^F} - \overline{\pi^F} - (1-\beta)/\beta}{rf}(-(1-\beta)/\beta + \overline{GY} + g^{pop}) \tag{A88}$$

The raio of consumption to GD follows from ~~equation~~ the GDP identity (M40):

$$CSN_t = 1 - \left(\overline{GSN} + \overline{IGSN} + (\delta + g^{AI} + \overline{GY} + g^{pop})\frac{(1-\beta)/\beta+\delta+rp}{(1-\tau)(1-t^P)(1-\alpha)} + \frac{\overline{\iota^F} - \overline{\pi^F} - (1-\beta)/\beta}{rf}(-(1-\beta)/\beta + \overline{GY} + g^{pop})\right) \tag{A89}$$

Using (M23) the steady state rate for labor income tax is:

$$t_t^W = \tau_0^W \tag{A90}$$

The steady state share of disposable income in GDP is (M48):

$$WSW_t = (1 - \tau_0^W - ssc)\bar{L}\frac{(1-\tau)(1+\overline{lol})\bar{L}}{\alpha} \tag{A91}$$

From (M25) follows the steady state lump sum tax:

$$T_t^{LS} = \left(\frac{1-\beta}{\beta} - \overline{GY} - g^{pop}\right)\bar{B} + \overline{GSN} + \overline{IGSN} + (\overline{TRW} - (\tau_0^W - ssc) + t^p)\bar{L}\frac{(1-\tau)(1+\overline{lol})\bar{L}}{\alpha} - t^P - t^C CSN_t \tag{A92}$$

The steady state growth rate of lump sum tax (M59):

$$GTAX_t = \overline{GY} + g^{pop} \tag{A93}$$

The steady state share of transfers to GDP (M57):

$$TRSN_t = \overline{TRW}\bar{L}\frac{(1-\tau)(1+\overline{lol})\bar{L}}{\alpha} \tag{A94}$$

The steady state growth rate of the exchange rate according to the purchasing power parity (M36):

$$GE_t = \bar{\pi} - \overline{\pi^F} \tag{A98}$$

The share of imports in GDP (M29):

$$IMSN_t = (1 - s^M)\left(\frac{P_t^C}{P_t^M}\right)^{\sigma^M - 1}(1 - NTBSN_t) \quad \text{(A99)}$$

The share of exports in GDP (M30):

$$EXSN_t = (1 - s^M)\left(\bar{E}^{\alpha^X s^M}\right)^{\sigma^X}\overline{YWY}^{\alpha^X} \quad \text{(A100)}$$

Using equations (M70) and (M71):

$$\pi_t^W = \overline{GY} + \bar{\pi} \quad \text{(A101)}$$

$$\pi_t^{WR} = \overline{GY} \quad \text{(A102)}$$

The steady state value of exogenous shocks is zero by definition:

$$u_t^C = u_t^\eta = u_t^{PX} = u_t^{PM} = u_t^{EX} = u_t^{CG} = u_t^{IG} = u_t^L = u_t^M = u_t^{AI} = u_t^F = u_t^{rp} = u_t^{TR} = u_t^W = 0 \quad \text{(A103)}$$

3.3.3 Calibration

An important problem in the case of such large scale models is the determination of model parameters. The model introduced here works with 126 parameters. In order to determine this amount of parameters, the information in even long time series is insufficient. In our case, the quarterly data between 1995Q1 and 2016Q4 are clearly not enough to ~~satisfyingly~~ identify all the parameters. Moreover, as usual in DSGE models, the system converges to a steady state in the long run which is determined by the parameters of the model. It is easier to obtain information from the data (trend-filtered time series) on the parameters describing the adjustment mechanisms towards the steady state, while the parameters which determine the steady state typically depend on the trend-characteristics of these time series. On the basis of this, it is common in the literature use basically three different approaches to identify the model parameters.

- Parameter identification with taking 'standard' or 'conventional' values from the literature.
- Parameter identification with 'calibration' which ties the parameter values to the data at hand but without the application of rigorous econometric techniques.
- Parameter identification through estimation when given parameters are determined by using econometric techniques in an integrated manner.

Following this distinction above, the standard methods in the literature and especially those applied for the QUEST model specification for the Eurozone, we determine part of the parameters by taking results from the original specification for the Eurozone, part of them by calibrating to the steady state and part of them by Bayesian estimation. In what follows, we report the parameter values which were taken from the original specification or calibrated.

3.3.3.1 Parameters taken from the original QUEST specification

As mentioned in the previous points, part of the parameters is used as specified in the version of the QUEST model estimated for the Eurozone. These parameters and their respective values are presented in Table 10.

Table 10 – Parameter values taken from the original (Eurozone) QUEST specification

Notation	Description	Value
α^X	The elasticity of exports to foreign GDP	0.5000
τ_1^{LS}	The reaction of lump sum tax on its deviation from target	$0.001*\tau_2^{LS}$
τ_2^{LS}	The reaction of lump sum tax on change in public debt	0.0040
$\overline{lol}$	The steady state share of overhead labor	0.0000
$\overline{YWY}$	The steady state value of the log ratio of foreign and domestic GDP	0.0000
ρ^{EX}	Persistence parameter, current account shock	0.9750
ρ^{lol}	Persistence parameter, overhead labor shock	0.9900
θ	Elasticity of substitution between labor types	1.6000
τ_1^W	The effect of output gap on labor income tax rate	0.8000
$\overline{ucap}$	The steady state capacity utilization	1.0000
φ	The elasticity of money stock to interest rate	0.4000
$\bar{\pi}$	Inflation target	0.005
$\overline{\pi^F}$	Foreign inflation target	0.005
δ	Depreciation rate for the private capital	0.025
δ^G	Depreciation rate for the public capital	0.0125
$\bar{B}$	The public debt to GDP target	2.4
γ_P	Parameter of the adjustment cost function for price	61.4415
h^L	Habit parameter in leisure	0.8089
τ_0^{IG}	The reaction of government investment (growth) on past change in the output gap	0.1497
κ	Parameter of the utility function	1.9224
ρ^{CG}	Persistence parameter, government consumption shock	0.2983
ρ^{rp}	Persistence parameter, physical investment risk premium shock	0.9182
ρ^{ucap}	Smoothing parameter in equilibrium capacity utilization	0.9517
σ_ε^η	The standard deviation of the markup shock	0.1500
σ_ε^{lol}	The standard deviation of the overhead labor shock	0.0048

In the case of parameters in Table 10, we employed them as used in the QUEST specification for the Eurozone. Some of these parameters need no modification due to their nature. Specifically the equilibrium capacity utilization and overhead labor rates belong to this category which comes into the model as straightforward normalizations. Substitution elasticity between labor types and the two persistence parameters we do not suggest a difference between the mechanisms in the reestimated version (with the augmented countries). The steady state value for the (log) ratio of foreign and domestic GDP means normalization on one hand and on the other it implies balanced growth rates in the domestic economy and in the rest of the world (note that this parameter defines the steady state and does not imply any restrictions on the adjustment mechanisms. The reaction parameters of the lump sum tax are of technical nature and their goal is to keep the public debt to GDP ratio close to its target level. The reaction of the labor tax rate is not estimated but set to a value used also in the Eurozone specification (where this parameter is not estimated as well). The elasticity of the money stock on interest rate has no real relevance because the money stock does not affect any other variables in the model. The domestic inflation target is set at 2%, a normal long run value rather than the explicit ECB target. Foreign inflation is set equal to the domestic long run value.

Parameters in Table 10 from γ_P on are estimated in the original specification for the Eurozone. In our reestimation we choose these parameters to set at their original values because they were not convincingly identifiable during estimation.

3.3.3.2 Steady state parameters

The second group of parameters consists of those values which were recalculated using the augmented data set, and are listed in Table 11.

Table 11 – Parameters calibrated using the augmented data set

Notation	Description	Value
α^G	The additive inverse of the production elasticity of public capital	0.0007
τ	Inverse of the elasticity of substitution between domestic varieties	0.0007
g^A	The steady state growth rate of TFP	0.0017
$\overline{GY}$	The steady state growth rate of per capita GDP	0.0022
α	The production elasticity of labor	0.5355
β	Discount factor	0.9965
$dgex$	The empirical trend of the export to GDP ratio	0.0082
$dgim$	The empirical trend of the import to GDP ratio	0.0075
$dgpm$	The empirical trend of the import price level	-0.0013
$dgpx$	The empirical trend of the export price level	-0.0015
$\overline{GSN}$	The steady state ratio of gov. consumption to GDP	0.1949
g^{pop}	The growth rate of population	0.0014
$\overline{GYW}$	The steady state growth rate of foreign GDP	0.0019
$\overline{IGSN}$	The steady state ratio of gov. investment to GDP	0.0313
$\bar{L}$	The steady state employment rate	0.8982
ssc	Social security contribution rate	0.3221
t^P	Tax rate of capital income	0.2434
$\overline{TRW}$	The steady state level of transfers (transfer to wage)	0.4485
t^C	VAT rate	0.2034
τ_0^W	Steady state rate of labor income tax	0.2434
g^{AI}	The steady state productivity growth rate of the intermediate sector	0.0006
ρ_1^{AI}	Persistence parameter, intermediate sector productivity shock, lag1	0.0000
ρ_2^{AI}	Persistence parameter, intermediate sector productivity shock, lag2	0.0000
ρ_3^{AI}	Persistence parameter, intermediate sector productivity shock, lag3	0.0000
ρ_4^{AI}	Persistence parameter, intermediate sector productivity shock, lag4	0.0000
σ_ε^{AI}	The standard deviation of the intermediate sector productivity shock	0.0039

The production elasticity of labor was set as the ratio of primary labor incomes to the GDP. The discount factor was set to match the real interest rate implied by the difference between nominal interest rates and inflation in the end of the sample period. In the case of the steady state parameters (rates) we used average values calculated for the estimation period. The trend parameters are obtained by fitting an exponential trend to the time series. Consider the following linear regression on the exponential trend of variable x:

$$\ln(x) = a + b \cdot t \tag{A104}$$

The trend of the original variable is thus: e^{bt}. The four trend variables are given according to this where we substitute the export to GDP, the import to GDP, the import deflator to GDP deflator and the export deflator to GDP deflator ratios respectively. Government consumption and investment to GDP ratios are determined as a time average of the respective values from the time series. Steady state employment is the average rate of employment (the ratio of employment to active population). The steady state labor tax rate is calculated as the time average of the ratio of labor tax revenues to labor income. The VAT tax rate is calculated as the time average of consumption and import tax revenues to total consumption while the steady state social security rate represents SSC revenues ratio to labor income. The ratio of transfers to wages is determined by the other revenues of the government (over consumption and investment expenses) to labor income ratio. The productivity growth process of the intermediate sector is reestimated for the augmented data set and we found a slight positive trend in productivity growth, however, the autoregressive coefficients were not significant, hence the reset calibration of these parameters.

The four parameters marked with grey in Table 1[illegible]e specific in the sense that they are set in a way that it is consistent with the TFP block of the GMR model. Specifically, the TFP block provides estimation for the expected long run growth rate of the aggregate TFP. This is then used to set the GDP growth rate which is consistent with the production function of the MACRO block. The markup parameter reflecting the elasticity of substitution between product varieties (τ) is set in line with the estimated parameter for employment in the patent equation of the TFP block, which represents agglomeration economies and contributes to increasing returns to scale in the SCGE block. Also, the power of public capital in the production function is linked to this elasticity to render the two production functions in the SCGE and MACRO blocks consistent. The details of setting these parameters are described in Appendix A.1.

3.3.3.3 Endogenous and technical parameters

Some parameters are a function of other parameters in the model (Table 12). One is the steady state growth rate in TFP, which is determined by equation (M15) on the basis of the steady state growth rates of employment and the two capital stocks as well as the production elasticities (see equation (A80)). The steady state capacity utilization (set to unity) determines the cost function parameter of capacity utilization adjustment on the basis of equation (M17). The parameter of the utility function (ω) is determined by the steady state employment and other parameters. Table 6 contains four additional parameters which serve technical purposes. Their role is to implement the required shock into the model when integrating it into the GMR framework. The standard deviations of these exogenous shock variables are set to zero.

Table 12 – Endogenous and technical parameters

Notation	Description	Value
γ_{U1}	Cost parameter of capacity utilization 1	0.0766
ω	Parameter of the utility function	0.9025
$\sigma_{\varepsilon}^{INV}$	The standard deviation of the investment growth shock	0.0000
$\sigma_{\varepsilon}^{CAP}$	The standard deviation of the private capital growth shock	0.0000
$\sigma_{\varepsilon}^{GCAP}$	The standard deviation of the public capital growth shock	0.0000
$\sigma_{\varepsilon}^{GB}$	The standard deviation of the government revenue shock	0.0000

3.3.4 Estimation

Those model parameters which are neither taken from the original setting, nor calibrated, are determined by estimation procedures. The estimation splits into two separate parts. First, we estimate the separate VAR model for the variables describing the evolution of the foreign sector (see model equations (M37)-(M39)) and second, the remaining parameters are estimated with Bayesian techniques. These estimation results are reported in what follows.

3.3.4.1 The database

In line with the estimation of the original specification for the Eurozone, the following quarterly time series are used for the estimation of the augmented version:

- Nominal short term interest rates
- Nominal effective exchange rate
- Nominal wage
- Employment
- Population in working age
- Household consumption
- Government consumption
- Total investment
- Government investment
- Imports
- Exports
- Gross National Product
- Deflator of the Gross Domestic Product
- Deflator of consumption goods
- Deflator of investment goods
- Deflator of imports
- Deflator of exports
- Government revenues from labor tax
- Government revenues from consumption taxes
- Government revenues from social security contributions
- Government transfers

For all of these time series we take the period between 1995Q1 and 2016Q4 as the basis of our estimations.

Part of the database (the time series for GDP, consumption, government and private investment, government spending, exports and imports) are extracted from the quarterly SNA tables of Eurostat. These data were seasonally adjusted using the X12-ARIMA method. The respective price indices were calculated from current and constant price data.

The time series for the rest of the world (quarterly inflation, GDP growth and interest rate data) is collected from Eurostat and OECD databases. Individual country data were weighted by Extra EU trade shares of the countries in the model database to obtain three time series for rest of the world GDP growth, inflation and interest rate. From the first two series we can recalculate (normalized) GDP volumes and the price index for the foreign sector.

For government data (transfers, consumption and income taxes as well as social security contributions) we also used data available from Eurostat. For the tax and social security rates, as they appear only as parameters, we calculate the average rates for the estimation period.

For the labor force (population) we use Eurostat data on the active population. For employment, we use direct employment data available from Eurostat.

The data are prepared in order to match with endogenous variables of the model. In accordance with the procedure used in the original setting, finally 17 observed data series are used corresponding to endogenous variables – these are listed in Table 13.

Table 13 – Observed endogenous variables

#	Notation	Description
1.	$\ln(CY_t)$	Consumption to GDP share (real)
2.	$\ln(E_t)$	Exchange rate (nominal)
3.	$\ln(GGY_t)$	Government consumption to GDP share (real)
4.	$\ln(IGSN_t)$	Government investment to GDP share (nominal)
5.	$\ln(ISN_t)$	Investment to GDP share (nominal)
6.	$\ln(L_t)$	Employment rate
7.	GY_t	Growth rate of per capita GDP
8.	$\ln(YWR_t)$	GDP to nominal wages ratio
9.	i_t	Domestic interest rate (nominal)
10.	π_t	Domestic inflation
11.	$\ln(P_t^M/P_t)$	Relative price of imports
12.	$\ln(P_t^X/P_t)$	Relative price of exports
13.	TRW_t	Transfer per capita to real wage ratio
14.	i_t^F	Foreign interest rate (nominal)
15.	π_t^F	Foreign inflation
16.	$\ln(YWY_t)$	Foreign GDP to domestic GDP ratio (nominal)
17.	g_t^{AI}	The growth rate of the productivity of intermediate goods

The observed variables listed in Table 13 can be logically calculated from the time series collected in our database. The raw data are transformed as follows: import and export prices are filtered with exponential trend, transfers are filtered by the transfers to wage ratio while the foreign and domestic GDP ratio is filtered with its own trend. The productivity growth of the investment goods sector can be given by the time change of the log deviation in investment deflator. Inflation is the log deviation of GDP deflator and other variables are transformed to per capita data dividing by the trend of working age population.

3.3.4.2 Macro processes of the foreign sector

The internal processes of the foreign sector are captured by three variables: foreign interest rate, inflation and GDP. We estimate a separate VAR model (see equations (M37)-(M39)) written for the cyclic components of these three variables. We used OLS estimation in line with the procedure in the original specification of the QUEST model. The standard deviations of the three shocks related to these three variables are also obtained from this estimation. The estimation results are summarized in Table 14.

Table 14 – Estimated parameters of the foreign VAR block

Notation	Description	Value
ρ^{i^F}	Smoothing parameter of foreign interest rate	0.8642
ρ^{i^F,π^F}	Effect of foreign inflation on foreign interest rate	-0.1151
ρ^{i^F,GY^F}	Effect of foreign GDP on foreign interest rate	0.1765
ρ^{π^F,i^F}	Effect of foreign interest rate on foreign inflation	-0.0328
ρ^{π^F}	Smoothing parameter of foreign inflation	0.3698
ρ^{π^F,GY^F}	Effect of foreign GDP on foreign inflation	0.1479
ρ^{GY^F,i^F}	Effect of foreign interest rate on foreign GDP	-0.0253
ρ^{GY^F,π^F}	Effect of foreign inflation on foreign GDP	-0.2204
ρ^{GY^F}	Smoothing parameter of foreign GDP	0.7031
$\rho^{GY^F,GY}$	Effect of the rate of domestic to foreign GDP foreign inflation on foreign GDP	-0.0001
σ_ε^{iW}	The standard deviation of the foreign interest rate shock	0.0053
σ_ε^{PW}	The standard deviation of the foreign inflation shock	0.0040
σ_ε^{YW}	The standard deviation of the foreign GDP shock	0.0045

3.3.4.3 Bayesian estimation

The remaining parameters (those which are not taken from the original specification, not calibrated and not belonging to the foreign VAR block) are estimated with Bayesian techniques.

In what follows, we specify the details of the estimation procedure and present the estimation results and diagnostic tests.

Estimation specification

First of all, we need to specify the prior distributions for the estimation. In this case we take the original specification of the QUEST model for the Eurozone as a reference point and used the prior distributions specified there. These distributions, in turn, are based in many cases on considerations regarded as standard in the literature. The prior distributions and their parameters are summarized in Table 15 which also shows the posterior means. The latter values are used during the model simulations.

Table 15 – Prior distributions and posterior means

Notation	Definition	Prior dist.	Prior mean	Prior std.	Posterior mean
γ_{U2}	Cost parameter of capacity utilization 2	Beta	0,0500	0,0240	**0,0230**
τ_0^{CG}	The reaction of government consumption (growth) on past change in the output gap	Beta	0,0000	0,0600	**-0,0722**
γ_I	Adjustment cost parameter of physical capital investments	Gamma	30,0000	20,0000	**37,7204**
γ_K	Adjustment cost parameter of physical capital investments	Gamma	15,0000	10,0000	**0,9492**
γ_L	Parameter of the adjustment cost function for labor	Gamma	30,0000	20,0000	**62,8442**
γ_{PM}	The weight of inflation indexing in the import markup	Gamma	30,0000	20,0000	**6,5719**
γ_{PX}	The weight of inflation indexing in the export markup	Gamma	30,0000	20,0000	**20,0406**
γ_W	Parameter of the adjustment cost function for wage	Gamma	30,0000	20,0000	**15,8993**
τ_{lag}^{CG}	The smoothing parameter of government consumption	Beta	0,0000	0,4000	**-0,4978**

τ_{adj}^{CG}	The reaction of government consumption (growth) on the deviation of G/Y from steady state	Beta	-0,5000	0,2000	**-0,0661**
h^C	Habit parameter in consumption	Beta	0,7000	0,1000	**0,6963**
τ_{lag}^{IG}	The smoothing parameter of government investment	Beta	0,5000	0,2000	**0,0539**
τ_{adj}^{IG}	The reaction of government investment (growth) on the deviation of GI/Y from steady state	Beta	-0,5000	0,2000	**-0,8723**
τ_{lag}^{i}	The parameter for interest rate smoothing	Beta	0,8500	0,0750	**0,9525**
ρ^C	Persistence parameter, consumption preference shock	Beta	0,8500	0,0750	**0,8493**
ρ^η	Persistence parameter, markup shock	Beta	0,5000	0,0200	**0,0810**
ρ^{PM}	Persistence parameter, import markup shock	Beta	0,8500	0,0750	**0,9362**
ρ^{PX}	Persistence parameter, export markup shock	Beta	0,8500	0,0750	**0,9200**
ρ^{IG}	Persistence parameter, government investment shock	Beta	0,8500	0,0750	**0,8776**
ρ^L	Persistence parameter, leisure preference shock	Beta	0,9500	0,2000	**0,9192**
ρ^{Lss}	Smoothing parameter in equilibrium employment	Beta	0,8500	0,0750	**0,9489**
ρ^M	The weight of past prices in import share	Beta	0,5000	0,2000	**0,1719**
ρ^X	The weight of past prices in export share	Beta	0,5000	0,2000	**0,2396**
ρ^F	Persistence parameter, foreign risk premium shock	Beta	0,8500	0,0750	**0,9133**
rf	The effect of external debt on foreign risk premium	Beta	0,0200	0,0080	**0,0286**
rp	Risk premium on physical capital	Beta	0,0200	0,0080	**0,0330**
ω^X	The share of domestic consumption	Beta	0,8000	0,0800	**0,8617**
sfp	The share of forward looking firms (final consumption goods)	Beta	0,5000	0,2000	**0,9343**
sfp^M	The share of forward looking firms (import goods)	Beta	0,5000	0,2000	**0,8629**
sfp^X	The share of forward looking firms (export goods)	Beta	0,5000	0,2000	**0,8950**
sfw	The share of forward looking households (wage setting)	Beta	0,5000	0,2000	**0,8628**
σ^C	Parameter of the utility function	Gamma	2,0000	1,0000	**2,9826**
σ^X	Foreign elasticity of substitution between domestic and foreign goods	Gamma	1,2500	0,5000	**1,8534**
σ^M	Domestic elasticity of substitution between domestic and foreign goods	Gamma	1,2500	0,5000	**2,1611**
slc	The share of liquidity constrained households	Beta	0,5000	0.1000	**0,3396**
τ_π^i	The reaction of the interest rate on inflation (Taylor rule)	Beta	2,0000	0.4000	**2,0245**
τ^{TR}	The effect of employment on transfers	Beta	0,0000	0,6000	**0,4883**
ρ^{TR}	Persistence parameter, transfers shock	Beta	0,8500	0,0750	**0,8804**
τ_{Y1}^i	The reaction of the interest rate on output gap (Taylor rule)	Beta	0,3000	0,2000	**0,2566**
τ_{Y2}^i	The reaction of the interest rate on output gap change (Taylor rule)	Beta	0,3000	0,2000	**0,0735**
$wrlag$	Smoothing parameter in wage setting	Beta	0,5000	0,2000	**0,3983**
σ_ε^C	The standard deviation of the consumption preference shock	Gamma	0,0500	0,0300	**0,0646**
σ_ε^{PM}	The standard deviation of the import price shock	Gamma	0,0200	0,0150	**0,0459**
σ_ε^{PX}	The standard deviation of the export price shock	Gamma	0,1000	0,0600	**0,0420**
σ_ε^{EX}	The standard deviation of the current account shock	Gamma	0,0050	0,0300	**0,0033**
σ_ε^{CG}	The standard deviation of the government consumption shock	Gamma	0,0500	0,0300	**0,0041**
σ_ε^{IG}	The standard deviation of the government investment shock	Gamma	0,0500	0,0300	**0,0352**
σ_ε^L	The standard deviation of the leisure preference shock	Gamma	0,0500	0,0300	**0,0878**
σ_ε^M	The standard deviation of the monetary policy shock	Gamma	0,0025	0,0015	**0,0007**
σ_ε^F	The standard deviation of the foreign risk premium	Gamma	0,0050	0,0030	**0,0032**

	shock				
$\sigma_{\varepsilon}^{rp}$	The standard deviation of the physical capital risk premium shock	Gamma	0,0050	0,0030	**0,0075**
$\sigma_{\varepsilon}^{TR}$	The standard deviation of the transfers shock	Gamma	0,0500	0,0300	**0,0037**
σ_{ε}^{W}	The standard deviation of the labor demand shock	Gamma	0,0500	0,0300	**0,1297**
σ_{ε}^{Y}	The standard deviation of the TFP shock	Gamma	0,0500	0,0300	**0,0075**

After the prior distributions are defined we used the Dynare software (Adjemian et al., 2011) to estimate model parameters on the basis of observed variables listed in Table 7. The estimation basically constitutes of two blocks:

1. In the first phase we use the Kalman-filter to determine the likelihood function. The maximum of this likelihood function gives an estimated mode of the posterior distribution which is the starting point of the second phase of the estimation. Generally this first step is done by some optimization procedures one generally used of which is the algorithm of Sims. Dynare provides several such algorithms but none of these was able to come up with a satisfying solution. In turn, we used an alternative in-built application of Dynare which provides an approximation to the maximum of the likelihood function on the basis of a Monte Carlo method. This option does not provide the maximum but robust enough to serve as a starting point for the second phase. In addition, this method calculates the optimal value of the jumping parameter for the Metropolis-Hastings algorithm (see below).
2. In the second phase we provide a numerical approximation to the posterior distributions using Markov Chain Monte Carlo method. In effect we simulate a sample of different parameter values the distribution (statistical characteristics) of which approaches that of the objective distribution (the posterior in our case) when the sample is large enough. A typical method is to use the Metropolis-Hastings algorithm which walks through the possible range of parameter values (defined by the prior distributions) and using the Kalman-filter it draws those parameter ranges which are the most likely (have high likelihood) for the given dataset.

In the second phase of the estimation procedure the size of the simulation is critical. For the final estimation we used a 300 thousand step MH algorithm in two blocks which gives a sample of 600 thousand parameter combinations. Using the jumping parameter determined in the first phase the acceptation rate moves between 30-35% during the MH algorithm which corresponds to the generally accepted rule-of-thumb. Two blocks are required to run convergence tests which helps in the identification of the parameters. To control for the 'burn-in' period of the MH algorithm (the period when the MCMC algorithm is not converging), the first 50% of the simulated 600 thousand units sample (in both blocks) is left out from calculating the posteriors and moments.

Estimation results

In what follows, we present the estimation results. We show the posterior distributions for the estimated parameters, the convergence tests and the in-sample forecasting performance of the model. Finally we give a brief comparison with alternative specifications.

Posterior distributions

Figures 4 present the posterior distributions (black line), the prior distributions (grey line) and the approximated posterior modes given by the first phase of the estimation procedure (dashed lines).[12] The layout of the posterior distributions can serve as a first impact on the quality of estimation results. If the posterior has the same shape and position as the prior we can infer that there is not enough information in the data to identify the given parameter (or, incidentally it may be the case that our prior choice was very accurate). Similarly, a posterior distribution with two or more modi signals that more parameter values are consistent with the model specification and the data. The signal of well identified parameters is the relatively narrow range for the distribution (relative to the prior), the smooth shape of the curve and a different mode compared to the prior (the last one is not a necessary condition as with an accurately chosen prior the modi can be the same).

As evidenced by the figures, most of the parameters can be regarded as well identified. Less well identified seems to be the persistence parameters of government investment and overhead labor

The less well identified parameters were left in the estimation on the basis of two considerations. First, a further condition for selection is the overall fit of the model (see later) and the fact that the persistence parameters are either set to zero during the simulations or we do not effectively use them in the absence of shocks.[13] In addition, convergence tests constitute a further selection criterion.

[12] Table 11 gives the concordance between the Dynare codes used in the diagrams and the parameter names used in the model description.
[13] Note that during the simulations only few shocks are used as described in a later section.

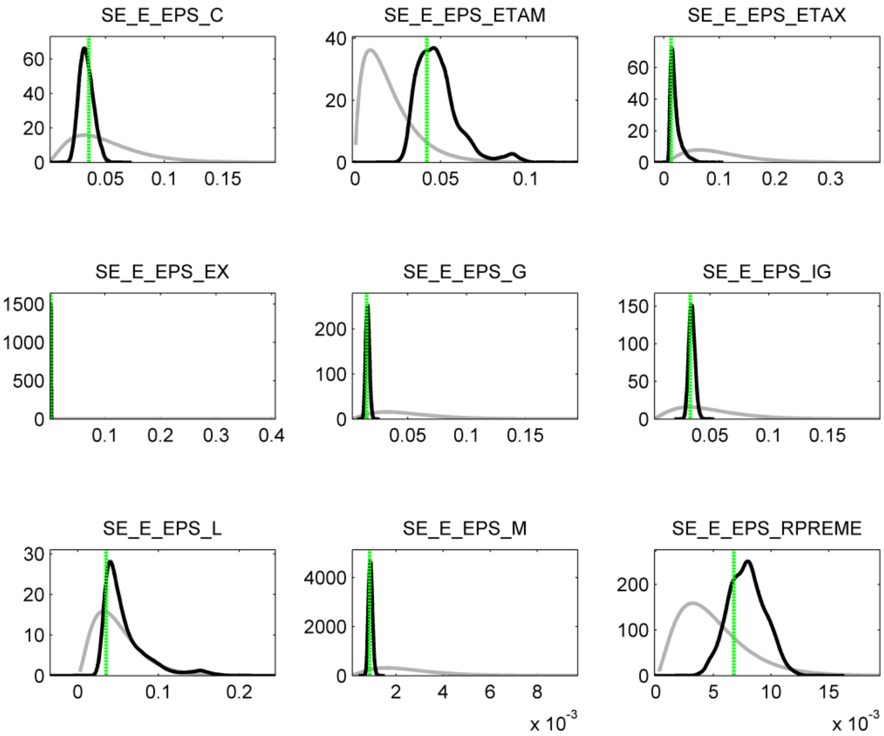

Figure 4a – Prior and posterior distributions

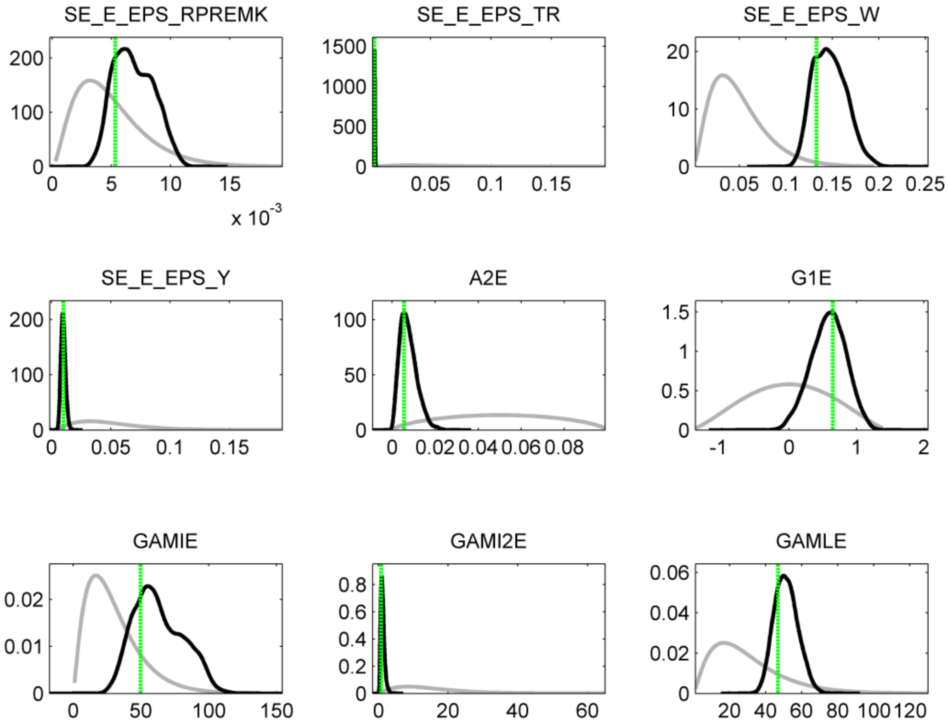

Figure 4b – Prior and posterior distributions

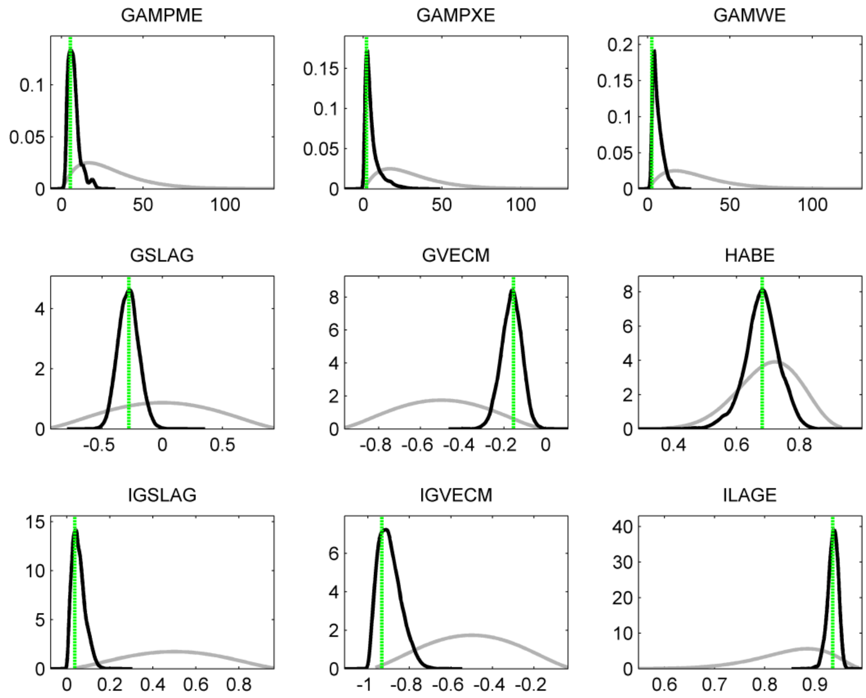

Figure 4c – Prior and posterior distributions

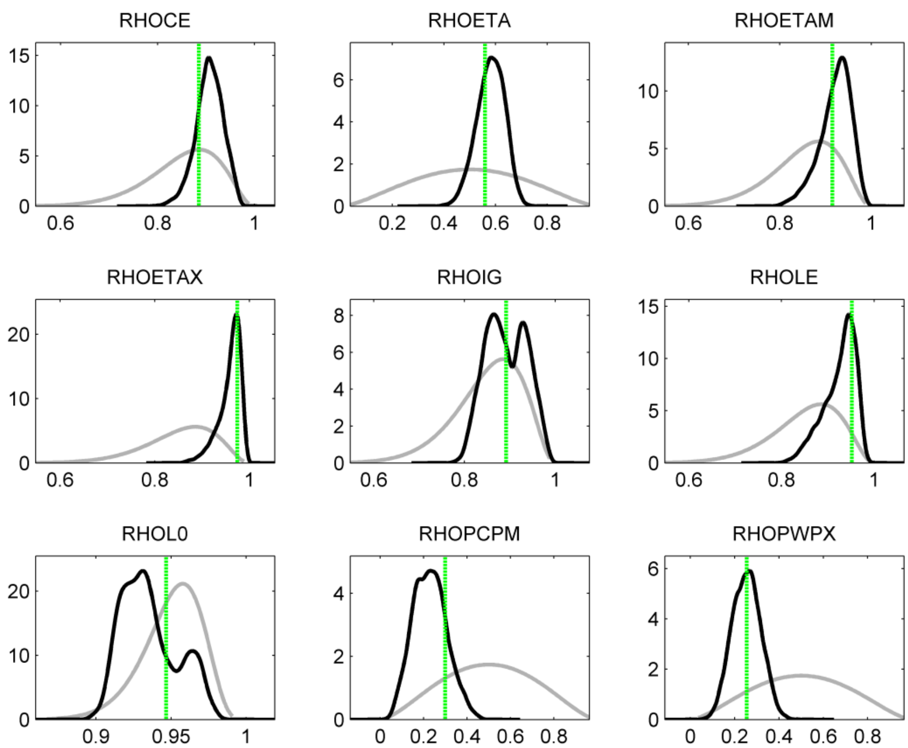

Figure 4d – Prior and posterior distributions

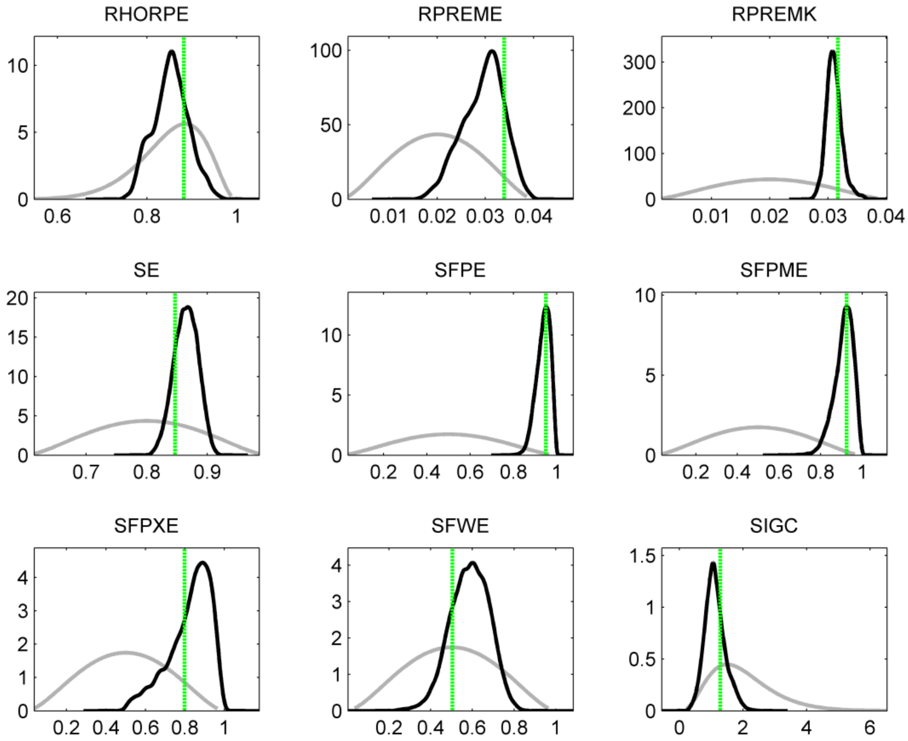

Figure 4e – Prior and posterior distributions

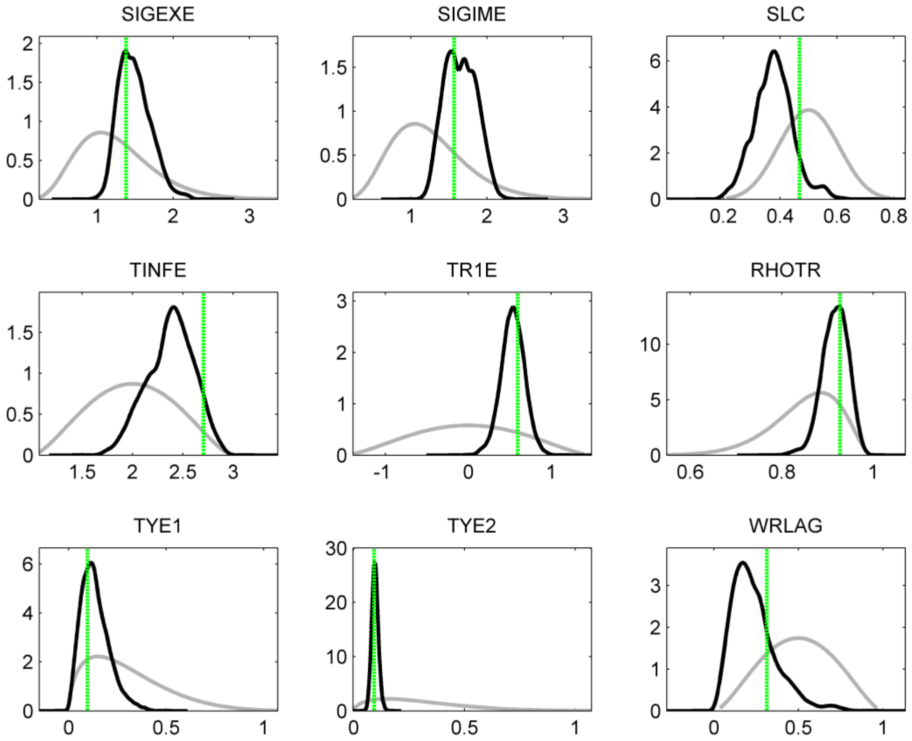

Figure 4f – Prior and posterior distributions

Table 16 – Correspondance between notations

Notation	Dynare notation	Notation	Dynare notation	Notation	Dynare notation
γ_{U2}	A2E	ρ^{PX}	RHOETAX	τ_{π}^{i}	TINFE
τ_0^{CG}	G1E	ρ^{CG}	RHOGE	τ^{TR}	TR1E
γ_I	GAMIE	ρ^{IG}	RHOIG	ρ^{TR}	RHOTR
γ_K	GAMI2E	ρ^{L}	RHOLE	τ_{Y1}^{i}	TYE1
γ_L	GAMLE	ρ^{Lss}	RHOL0	τ_{Y2}^{i}	TYE2
γ_P	GAMPE	ρ^{M}	RHOPCPM	$wrlag$	WRLAG
γ_{PM}	GAMPME	ρ^{X}	RHOPWPX	σ_{ε}^{C}	E_EPS_C
γ_{PX}	GAMPXE	ρ^{F}	RHORPE	$\sigma_{\varepsilon}^{\eta}$	E_EPS_ETA
γ_W	GAMWE	ρ^{rp}	RHORPK	$\sigma_{\varepsilon}^{PM}$	E_EPS_ETAM
τ_{lag}^{CG}	GSLAG	ρ^{ucap}	RHOUCAP0	$\sigma_{\varepsilon}^{PX}$	E_EPS_ETAX
τ_{adj}^{CG}	GVECM	rf	RPREME	$\sigma_{\varepsilon}^{EX}$	E_EPS_EX
h^{C}	HABE	rp	RPREMK	$\sigma_{\varepsilon}^{CG}$	E_EPS_G
h^{L}	HABLE	ω^{X}	SE	$\sigma_{\varepsilon}^{IG}$	E_EPS_IG
τ_{lag}^{IG}	IGSLAG	sfp	SFPE	σ_{ε}^{L}	E_EPS_L
τ_{adj}^{IG}	IGVECM	sfp^{M}	SFPME	$\sigma_{\varepsilon}^{lol}$	E_EPS_LOL
τ_{lag}^{i}	ILAGE	sfp^{X}	SFPXE	σ_{ε}^{M}	E_EPS_M
τ_0^{IG}	IG1E	sfw	SFWE	σ_{ε}^{F}	E_EPS_RPREME
κ	KAPPAE	σ^{C}	SIGC	$\sigma_{\varepsilon}^{rp}$	E_EPS_RPREMK
ρ^{C}	RHOCE	σ^{X}	SIGEXE	$\sigma_{\varepsilon}^{TR}$	E_EPS_TR
ρ^{η}	RHOETA	σ^{M}	SIGIME	σ_{ε}^{W}	E_EPS_W

ρ^{PM}	RHOETAM	slc	SLC	σ_{ε}^{Y}	E_EPS_Y

Convergence tests

A further test on the quality of estimation results is whether the metropolis-Hastings algorithm converges, so that to what extent the resulting posterior distributions confines with the underlying true distribution. A widely used test for convergence is the diagnostics developed by Brooks and Gelman (1998) which is based on within and between variances. To calculate the test, in each iteration of the MH algorithm we calculate the within variances in each block (then taking their average) and the between variance among blocks. The condition of convergence is that between variance goes to zero (i.e. the average values of the different blocks converge to each other) while the within variance stabilizes. These statistics can be calculated for the estimated parameters separately, but an overall value can also be constructed. In addition, the tests can be calculated for any moment of the posterior distribution. The overall convergence test of our estimation is shown in Figure 5.

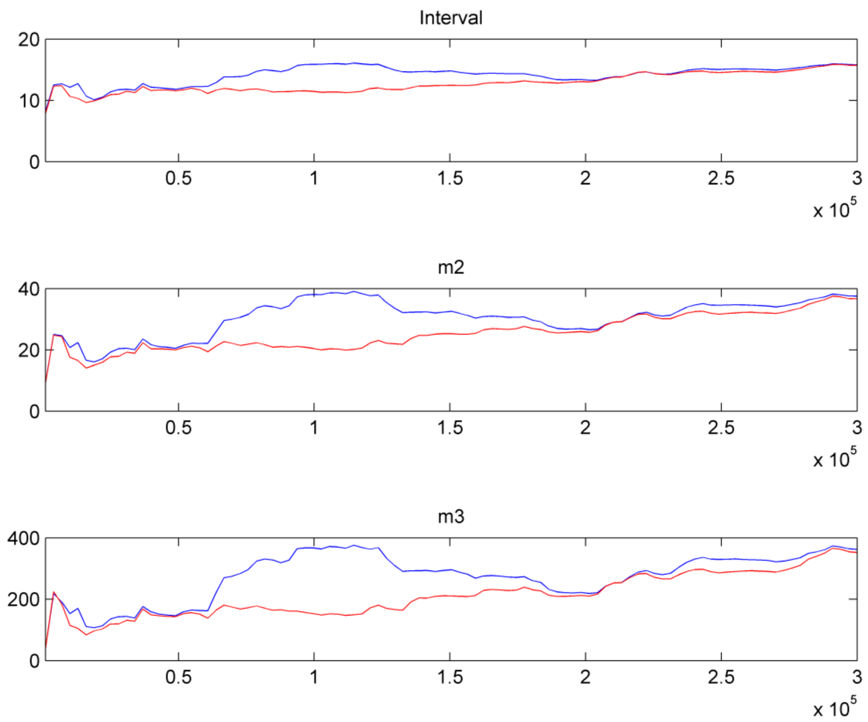

Figure 5 – Overall convergence diagnostics

In the case of the convergence test generated by Dynare the red (lower) line represents the within variance while the blue (upper) shows the sum of between and within variances. As a result, converging lines mean convergence among the blocks and stabilizing lines show convergence in the distribution as a whole. The three panels show the first, second and third moment statistics respectively. According to the figure, we can infer that on average the parameters are characterized by good convergence, between variance disappears while within variance stabilizes, although higher moments show less perfect convergence.

In addition to the overall statistics it is also important to examine the individual convergence tests of the estimated parameters. These are shown in Figures 6. The convergence tests are generally acceptable for most of the parameters, unsatisfying results mostly accord with those parameters for which the posterior distributions sign a less strong identification.

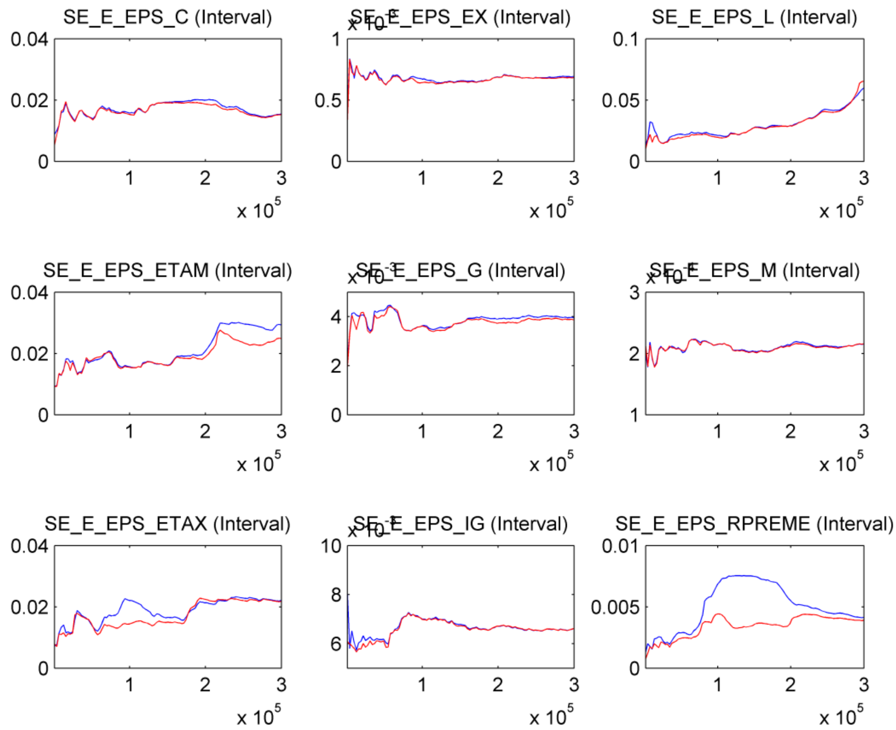

Figure 6a – Convergence tests for separate parameters

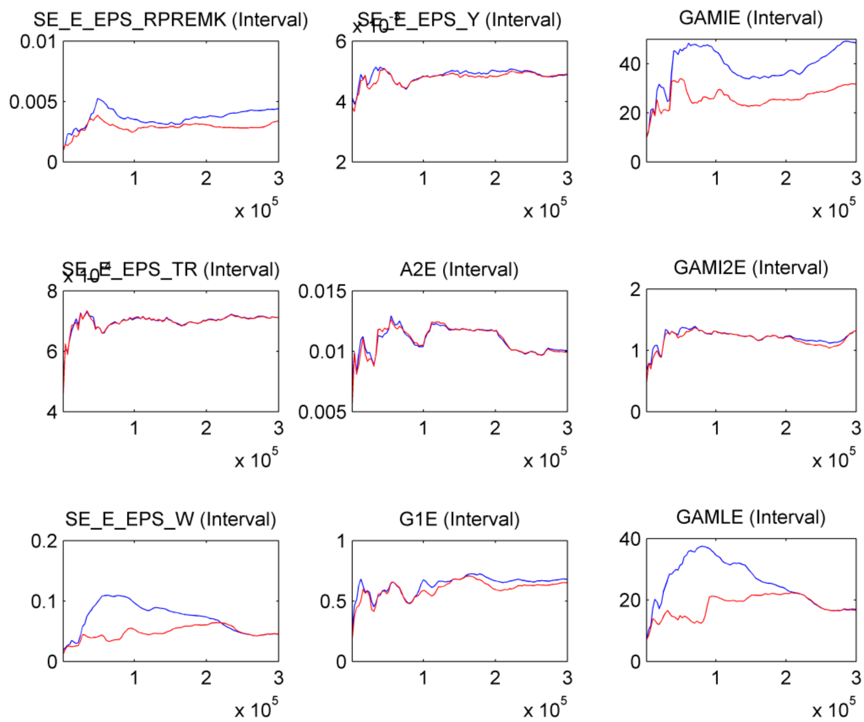

Figure 6b – Convergence tests for separate parameters

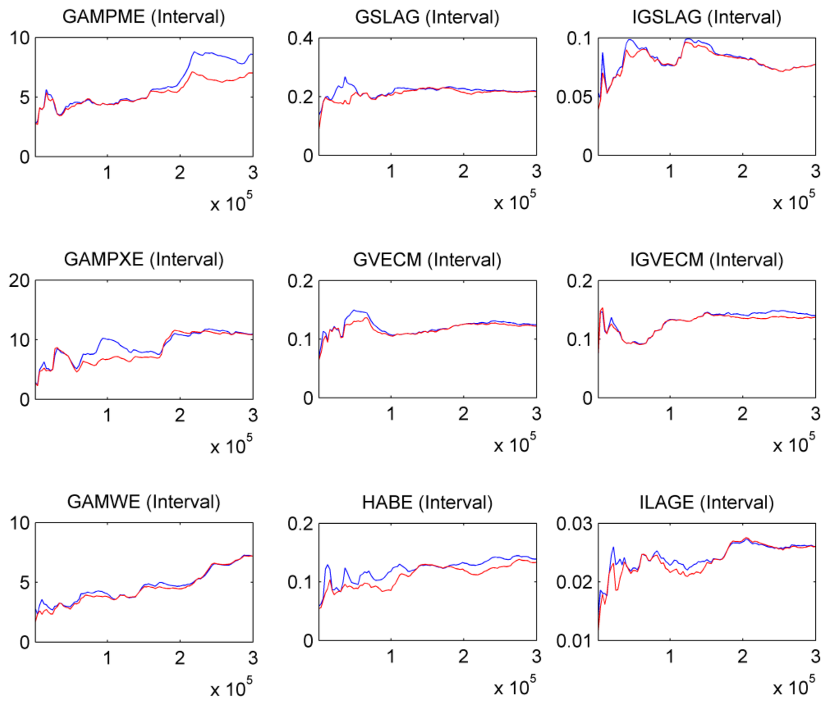

Figure 6c – Convergence tests for separate parameters

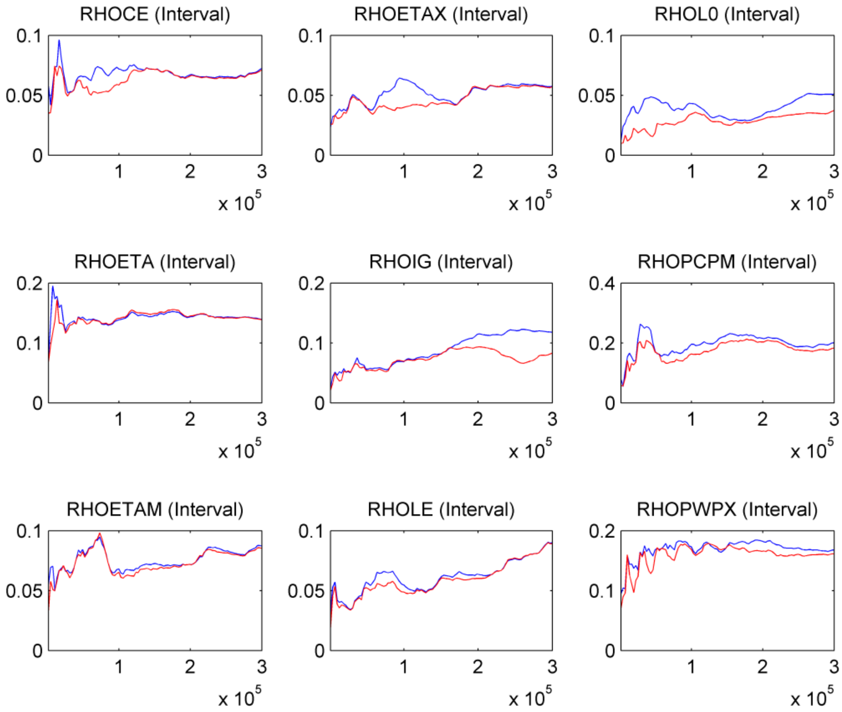

Figure 6d – Convergence tests for separate parameters

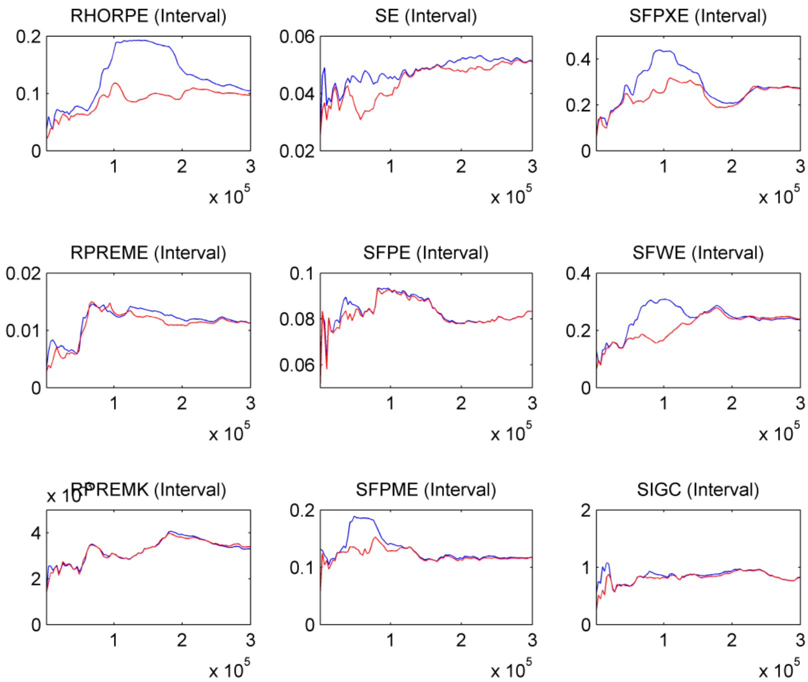

Figure 6e – Convergence tests for separate parameters

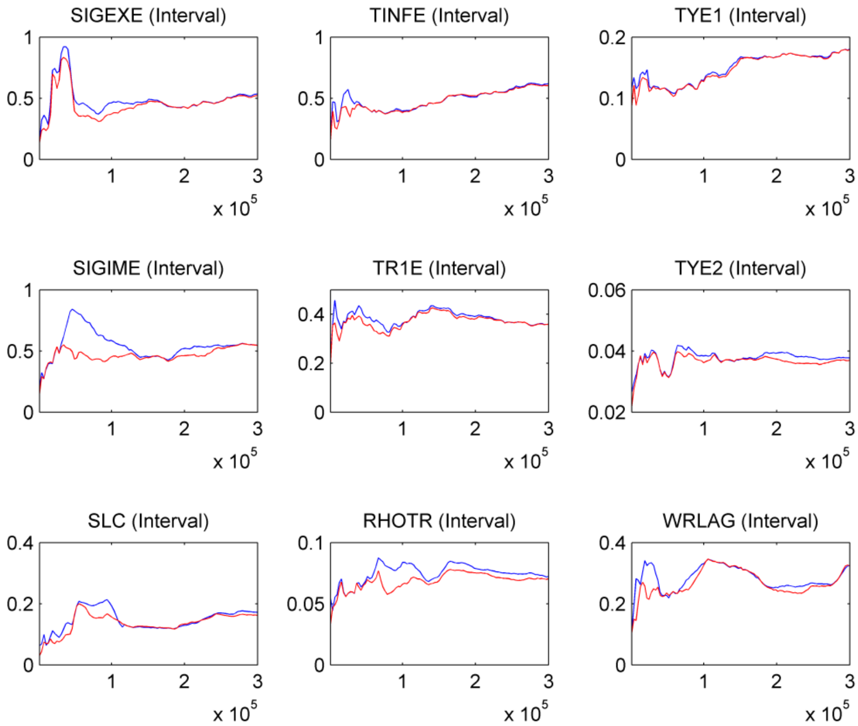

Figure 6f – Convergence tests for separate parameters

In-sample forecast

Beyond the individual evaluation of parameter estimates, a good test for the fit of the model is to examine its in-sample forecast performance. In order to do this, we prepared a one period ahead forecast with the Kalman filter for the observed endogenous variables. The nine most important of these are shown on Figure 7. The solid line marks the observed time series (after the transformations discussed previously) while the dashed line is the one period ahead forecast. The results show good in sample forecast performance in all cases.

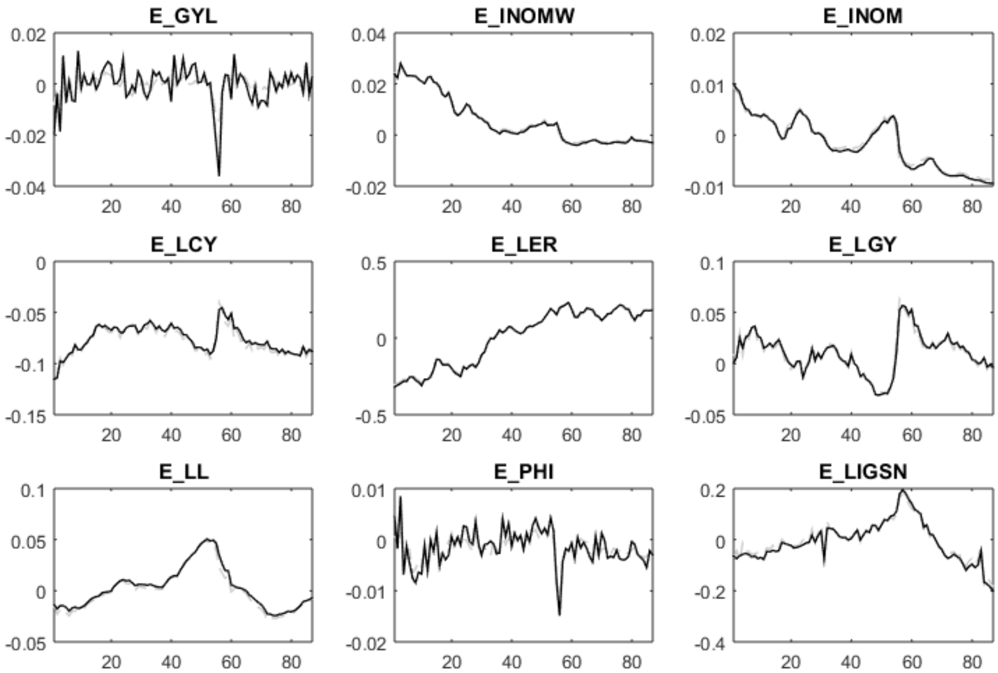

Figure 7 – In-sample forecast for some endogenous variables

Comparison of alternative specifications

The general fit of the estimated models can be described with the marginal density value: the ratio of these values calculated for two different specifications is called the Bayes factor and shows the extent to which a specification is more likely than another given the data. Table 16 summarizes two specifications.

Table 17 – Model fit for different specifications

Specification	*1.*	*2.*
Number of estimated parameters	63	54
MH iterations	300	300
Marginal density	***5497***	***5506***

In the first specification we estimated all parameters (63) which were also estimated in the original specification of the model for the Eurozone. In the second specification we set those parameters to their original values from the Eurozone specification the identification of which seemed problematic. This way we left 54 parameters in the Bayesian estimation procedure and the resulting model gives the better fit.

Impulse responses

As the simulation of the model is implemented through running impulse responses, it is important to examine the reaction of some focal variables to shocks. In Figures 8 the reaction of four endogenous variables (employment – E_LL, GDP growth – E_GYL, the growth rate of private capital stock – E_GK and the growth rate of public capital stock – E_GKG) are depicted in response to shocks to the TFP growth rate (Figure 8a), to government consumption (Figure 8b) and government investment (Figure 8c). The figures show the deviation of the respective variables from their steady state values while the grey area marks the confidence interval.

On the vertical axes of the impulse responses (in line with the in-built features of Dynare but differing from the standard interpretation) absolute and not percentage deviations are depicted. If we take the endogenous variable x_t the steady state value of which is x^*, then the impulse response is $irf_t = x_t - x^*$. The impulse responses show in each case the fade-out of a one standard deviation shock. In the case of the TFP this is 0,0075, for the government consumption it is 0,0041 and for government investment it is 0,0352. In each case the model uses quarterly growth rates so the magnitudes of the shocks are to be interpreted according to this.

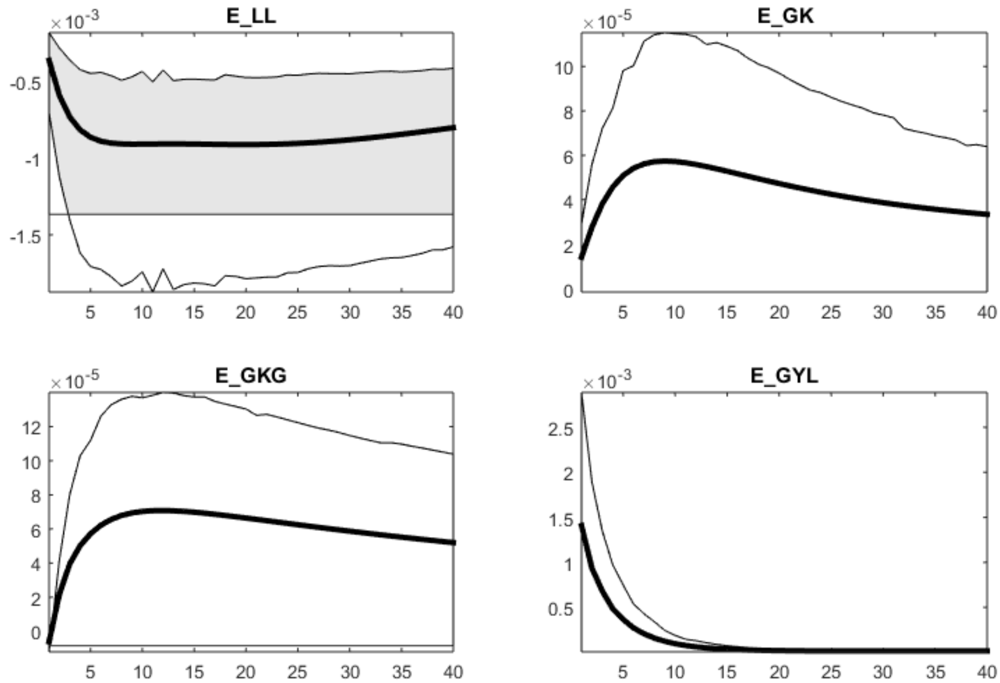

Figure 8a – The reaction of output variables on a shock to TFP growth

A shock to the TFP has a positive effect on GDP growth (which results in a positive shift in GDP levels). According to equation (M87) the growth rate of TFP follows a random walk with drift the persistence of which is zero. This drives the relatively rapid fade-out of the TFP shock. However, it is important to note that the persistence of the TFP shock is endogenized by the other two model blocks (TFP and SCGE blocks), and the macro model only simulates the macroeconomic spillover effects of these exogenous shocks. However, it is less visible on the figure that after the relatively large jump in the beginning, the GDP growth rate persistently remains over the steady state level for a long while.

The employment effect is negative, which is a general reaction in DSGE models. The reason is that the productivity growth leads to price decreases but due to staggered price setting prices change slowly which makes it optimal for firms to hire less labor. However, this negative effect is balanced in the long run by the increasing labor demand stemming from increased productivity. Public and private capital stocks react positively to TFP shocks with private capital having a persistent effect in growth and the effect on public capital fading out over time.

It is worth mentioning that the sharp contrast between the fade-out of the GDP and the other three variables is misleading from the picture. It happens that GDP growth is directly and strongly affected by the TFP shock as it enters into the production function. After a sharp decrease, though, GDP growth remains over the steady state for almost the entire period depicted here with a deviation from the steady state corresponding in magnitude to the deviations of the other three variables.

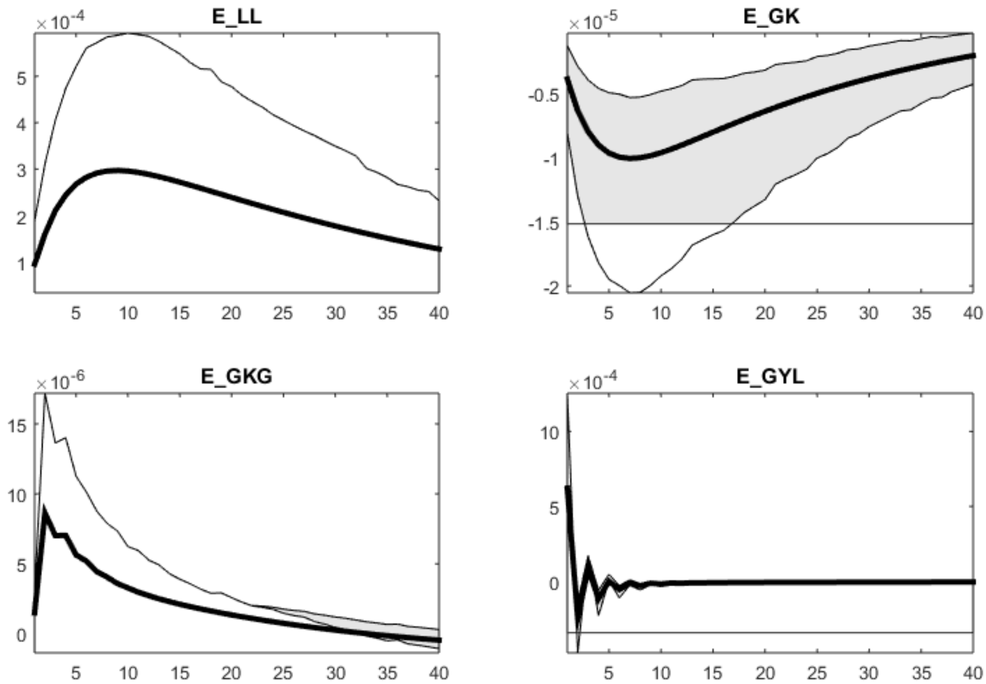

Figure 8b – The reaction of output variables on a shock to government consumption

The shock to government consumption generates a positive employment effect throughout the response horizon, in magnitude similar to that of the TFP shock while its effect on GDP growth is minimal and shows cyclical properties. Public capital moves in a positive direction while due to the crowding out effect private investment decreases.

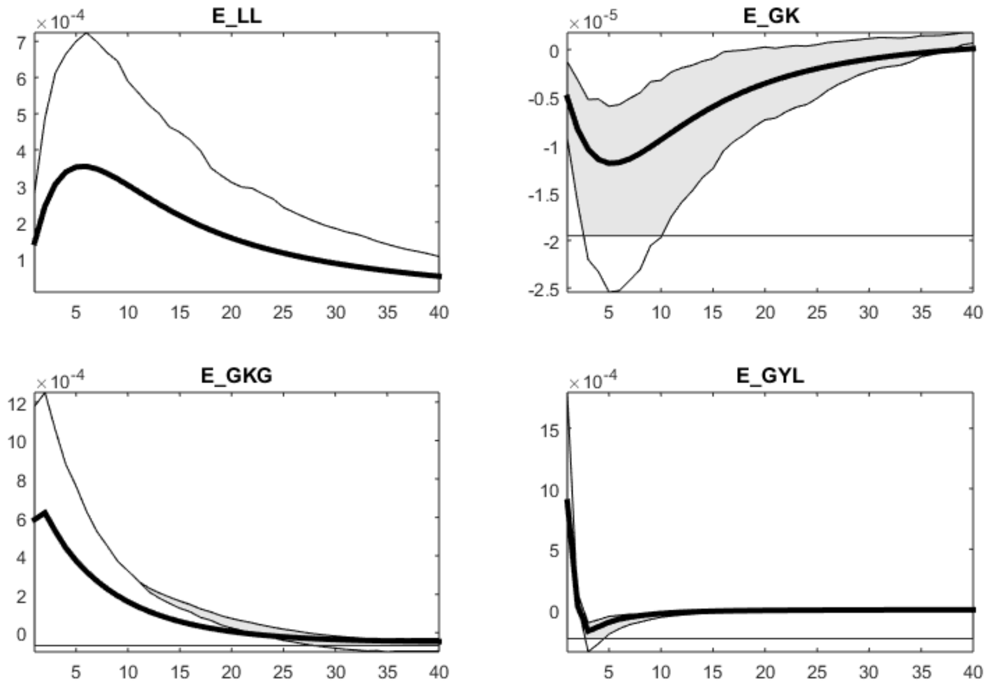

Figure 8c – The reaction of output variables on a shock to government investment

In the case of a shock to government investment we observe overall a more positive (but in the beginning of the period still negative) effect while the employment increases to the same extent as for government consumption. The effect on GDP growth in magnitude is almost the same as in the case of government consumption, but the cyclical tendency is less prevalent.

It is interesting to see the effect of shocks on levels rather than rates. It is true that only the TFP shock has a persistent level effect on GDP and a smaller effect on capital stocks. Although the government consumption and investment shocks give paths different from steady state, this difference is small (around 0,5% at the most extreme point) and after 50 periods converge back to the steady state path. In the case of employment, the impulse response shows levels by definition.

3.3.5 Integrating the MACRO block into the rest of the GMR model

Tayloring the macro block into the GMR model (in practice with the SCGE block) means basically three steps. The first step is an input interface through which the macro block receives the inputs, the second step is running the macro block which means calculating impulse responses on the input shocks and the third step is providing the SCGE block with the time series generated by the impulse responses.

3.3.5.1 Inputs to the macro block

The macro block requires five time series as an input. These time series are as follows:

1. Time series of TFP levels
2. Time series on shocks (additions, policy interventions) to government consumption constituting of spending on education, R&D support and other demand side stimuli.

3. Time series on shocks (additions, policy interventions) to government investment which corresponds to infrastructural investment.
4. Time series on private investment support.
5. Time series on scheduled repayment of private investment supports.

These time series are available from the SCGE block on an annual frequency, so the input interface of the macro block first converts them into quarterly values and then generates the necessary shock variables from these series which are then the direct inputs to the model. From the regional TFP time series generated by the SCGE block we calculate the aggregate TFP levels as a weighted average of regional TFP values, where the weight is the population of the regions. From these aggregate TFP levels we calculate annual growth rates of the aggregated TFP. The remaining four inputs are simply summed up across regions to generate the aggregate level inputs to the MACRO block.

In the case of the TFP, annual growth rates are converted to quarterly in a way that quarterly rates sum up to annual rates. Then, quarterly growth rates are related to the steady state growth rate in order to obtain those shocks which are the inputted to the macro model.

In the case of government consumption and investment we also split annual data into quarters, assuming even distribution within years. At the same time we have to take into account that government consumption and investment enters into the macro model through growth rates (see equations (M20 and (M21)), so in each quarter we have to convert additional consumption and investment into growth rates. In order to do this we calculate the volume of government investment and consumption throughout the model run and we get the required shocks comparing additional interventions to these volumes.

In the case of private investment support we also split annual interventions to quarters evenly, which (as in the case of government consumption and investment) is inputted into the model after converted into additional growth rates. Repayments of investment support are accounted for as (negative) transfers to the government budget.

3.3.5.2 Running the MACRO model

Running the macro model basically means applying the reduced dynamic matrix equation in (A44). This matrix equation uses transition matrices determined by model parameters with which it is able to generate the time path of endogenous variables as a response to arbitrary shocks to the system. As a result, using the exogenous shock variables (both those originally in the model and those added here to implement policy interventions) we can simulate the effect of government interventions and TFPs given as inputs and we can trace the resulting macroeconomic processes for the endogenous variables of the model.

Implementation of the shocks in the model is done according to the following mechanisms:

- The growth rate of TFP is given by equation (M87) with the help of the exogenous shock variable ε_t^Y. According to this equation TFP follows random walk with drift where the trend is given by the steady stat growth rate of the TFP. Subtracting the steady state TFP growth from the TFP growth rates coming as inputs we obtain that value for ε_t^Y which acts as a shock to the system.

- Government consumption can be influenced by the variable u_t^{CG} in equation (M20). As written earlier, this equation works with growth rates so the additional quarterly consumption inputs (given in levels) are converted into additional growth rates using the value of the consumption expenditures of the previous quarter in order to obtain the required value for u_t^{CG}. As u_t^{CG} is a persistent exogenous variable in the original model setting, which is driven by equation (M95) and shock ε_t^{CG} in it, the persistence parameter ρ^{CG} in equation (M95) is set to zero during the simulations so that we can simulate the clear effect of interventions.
- Simulating government investments is analogous to that of government consumption. Here, we implement the interventions through the exogenous variable u_t^{IG} in equation (M21) as additional growth rate. Similarly to consumption, in equation (M96), driving u_t^{IG} we set the persistence parameter ρ^{IG} to zero. In addition, the higher growth rate of government investment must be inputted also into the growth rate of public capital. This is done through the exogenous shock ε_t^{GCAP} in equation (M43).
- Private investment subsidies are implemented analogously to government investment. The exogenous shock variable ε_t^{INV} in equation (M47) influences the growth rate of private investment whereas the exogenous shock variable ε_t^{CAP} in equation (M42) influances the growth of private capital stock in accordance with the interventions.
- Increasing only the expenditure side of the government budget (consumption and investment) we would observe an additional deficit leading to an increase in public debt. However, the financing source of these expenditures are given in principle, but not accounted for in the model structure. As a result, we have to implement an additional element on the revenue side of the government budget to include the financing of these expenditure elements. This issue is handled through the exogenous shock variable ε_t^{GB} added to equation (M25). As this equation is given relative to the nominal GDP, we have to trace the nominal GDP level in each period and using this value we can determine that value for ε_t^{GB} which balances the budget expenditures.
- As these revenues are financed from the foreign sector, we also adjust the current account to GDP ratio with the variable ε_t^{GB}. In this setting, we assume synchronized dynamics in the resources and the expenditures of the government budget and as a result, the adjustment of the current account is mostly of technical nature.
- Possible repayments are implemented as negative transfers flowing from the private sector to the government, using the exogenous variable u_t^{TR} in equation (M22). Repayments have to be included here as a ratio to wages so the wages are also traced during the simulation run and we can calculate the value of the shock variabla on the basis of this information. As the variable u_t^{TR} is persistent in the model, we set the persistence parameter ρ^{TR} in equation (M103) to zero during simulation run.

Running the (A47) recursive system of equations with the shock variables calculated according to the principles given above, as a result we obtain the time paths of the endogenous variables.

3.3.5.3 Outputs from the MACRO block

The simulated time series of endogenous variables form the macro model is used by the SCGE block. However, only few of the 104 endogenous variables are used: these are the time series for GDP, employment, and government consumption and investment. These outputs are generated by the macro model in a way that for the first year the values are unity and the relative changes are reported for each consecutive year. We use quarterly growth rates for these four variables to calculate output, the cumulative annual growth rates are used to obtain the indices for the output variables for each year.

The macro block generates as output further time series which are not used by the SCGE block. These are the consumption of households, unemployment rate and the deficit to GDP ratio. Household consumption is also given as an index with the first year normalized to one and the other two values are reported naturally in percentages. Due to its special nature, we separately discuss the unemployment rate in what follows.

3.3.5.4 Unemployment

As a general equilibrium model, the macro block does not contain a direct measure for unemployment as the markets, including the labor market, clear in every period. As a result, there is no explicit unemployment in the model, so we can only provide an approximation to it. This approximation is made possible by the variable L_t^{ss} describing equilibrium employment (see equation (M26)). We assume that this value corresponds to labor market equilibrium which is characterized by the natural rate of unemployment. As the variables L_t^{ss} and L_t are employment rates, we can write that

$$F_t = AP_t L_t$$

$$MK_t = AP_t L_t^{ss}$$

where F_t is employment, AP_t is active population and MK_t is the absolute values of labor supply. From these it follows that

$$\widehat{UR}_t = 1 - \frac{F_t}{MK_t} = 1 - \frac{L_t}{L_t^{ss}}$$

where $\widehat{UR}_t$ is the unemployment rate. As L_t^{ss} is interpreted as the employment rate corresponding to the natural rate of unemployment, if $L_t = L_t^{ss}$, or equivalently $\widehat{UR}_t = 0$, then unemployment equals the natural rate. As a consequence, $\widehat{UR}_t$ gives the deviation of unemployment from the natural rate, so for unemployment we can write the following formula with UN_t denoting the natural rate:

$$UR_t = UN_t + \widehat{UR}_t$$

4 Policy simulations

In this part of the report we provide a brief summary of a simulation exercise we have set up in order to illustrate the potential use of the GMR-Europe model in evaluating smart specialization policies. The simulation we use focuses on entrepreneurship and assumes an increase in the entrepreneurial climate of European regions as measured by the REDI index. The REDI index is part of the TFP block of the GMR-Europe model (see section 3.1 for details) i.e. an improvement in the entrepreneurial climate of a region is reflected in its productivity which then contributes to economic development in that region, the latter also interacting with other regions through trade and factor mobility. First, we give a short account of the simulation setup and key variables of interest and then we sum up the experiences from the simulation exercise.

4.1 Simulation setup

The goal with this simulation is to illustrate the potential use of the GMR-Europe model in evaluating policies targeting entrepreneurship. Entrepreneurship enters the model through the REDI index in the TFP block, by accounting for the entrepreneurial climate or ecosystem of regions. The REDI index gives a score for every region in the model which reflects the relative development level of the given region's entrepreneurial ecosystem/climate (see the details in section 3.1.3). This index enters the TFP equation of the TFP block, so improvements in the entrepreneurial ecosystem of a region are assumed to contribute to the overall productivity of the region which then affects economic variables over time in interaction with developments in other regions.

In the simulation setting used here, we track the effects of changes in the REDI index. In a more formal way, we follow the strategy below:

1. We take the baseline REDI scores of the model. The base year is 2012 and the baseline of the TFP block goes along empirically fitted trends from 2012 to 2030, which means that in the baseline model runs the REDI score of every region proceeds along a trend line derived from the observed data.[14]
2. For every region, we calculate the average of the baseline REDI scores over the simulation years (2012-2030) and take 1% of these average scores as a shock.
3. This 1% shock to the REDI index is applied in every region in a way that the REDI index is increased from its baseline value to a 1% higher value through the first 5 years of the simulation.
4. Every region gets this 1% shock in the REDI index and we trace the effect of these shocks on regional TFP (total factor productivity) and GDP levels as well as aggregate country-level versions of these variables.

Of course, focusing on the REDI index provides a bird-eye-view approach on entrepreneurial policies. We can interpret the idea behind these simulations as what happens if the entrepreneurial climate/ecosystem improves in the regions in question. We use this approach for illustrative purposes, but also emphasize that the detailed structure of the REDI index (see section 3.1.3) allows

[14] See section 3.1.5 for the details of the TFP block setup and the baseline trends.

the model to account for more detailed approaches in this respect. Overall, these simulations reflect potential effects of policies which are capable of improving the entrepreneurial ecosystem of a region by adjusting either of the pillars behind the REDI index. More elaborate simulations could analyze more specific policies of course.

4.2 Simulation results

~~In this section we briefly discuss the results of the simulation set up before.~~ Although the model is capable of tracking many regional and aggregate level variables, we display the effect of the policies (shocks) on TFP (total factor productivity) and GDP. In both cases we present the percentage deviation of the simulated TFP/GDP values from their baseline levels. As a result, the diagrams reflect the percentage impact of these policies: to what extent TFP and GDP would be higher/lower if the policy is in effect compared to the no-intervention (business as usual) case.

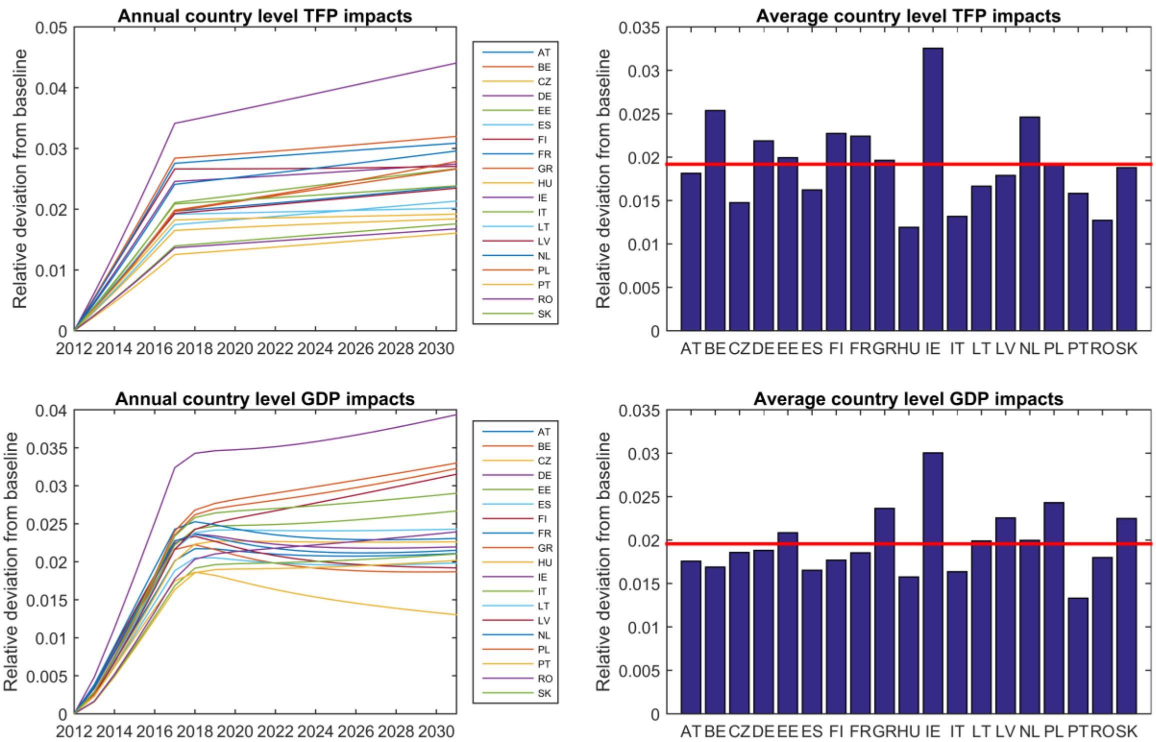

Figure 9 – Country level impacts of 1% shocks to REDI on TFP and GDP

In Figure 9 we summarize country level results of the simulations. On the left hand side the evolution of country level impacts can be observed for both output variables. On the right hand side the time-averages of these impacts are depicted for countries. The red line shows the EU-average impact. It is clear from the picture that 1% improvement in the entrepreneurial climate in every region leads to a 2% increase in GDP and productivity on average. The GDP impact is slightly higher, but the productivity and GDP effects go very close to each other which is not surprising as in the simulations the entrepreneurship policy exerts its effect through enhancing regional productivity. The top left diagram shows that the positive development in the entrepreneurial environment of regions positively affects the productivity levels in all countries. However, there are differences in the magnitude of this effect. While Ireland benefits the more from this policy (exceeding 4% productivity gain from the policy at the end of the simulation period), Hungary seems to be the worst performing from this respect (less than 2% productivity gain).

As mentioned before, GDP impacts follow quite close the TFP impacts, however, as seen from the bottom-left diagram, there are considerable qualitative differences. In some countries, although the overall effect of the policy is positive, after the 'lifting' power of the policy (first 5 years) dies out, the impacts tend to decrease compared to the peak year. The most visible this effect is in Portugal, but similar tendencies can be seen in Spain or Belgium. This effect is due to the complex mechanisms within the GMR model where productivity growth and the resulting economic development affects and feed back to that of other regions through trade and factor mobility. These feedback mechanisms may result in out migration or capital flight which negatively affects the growth of some regions.

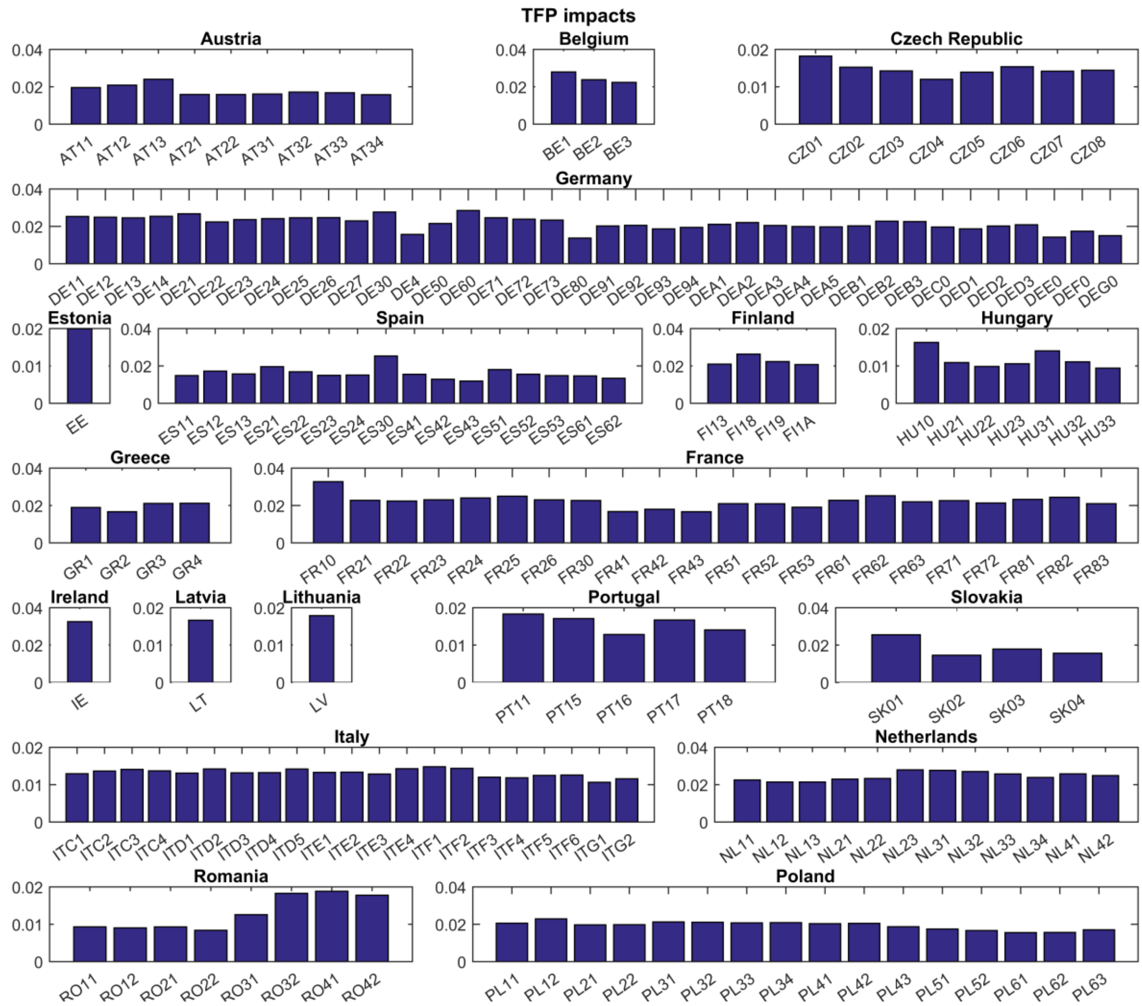

Figure 10 – Regional impacts of 1% REDI shocks on regional TFP levels

Figures 10 and 11 show the regional breakdown of the simulated impacts. As it can be seen, GDP impacts (Figure 11) follow the productivity impacts (Figure 10), but there are considerable differences between regions. In most of the cases we see that central (more developed) regions gain more from these policies. Also, the complex interaction mechanisms in the GMR-model are visible at the regional level marked by these significant differences in regional impacts and especially in favor of central regions: due to their economic weight these regions are able to attract production factors in the long run, therefore policy interventions in less developed regions seem to partially contribute to the development of other regions as well.

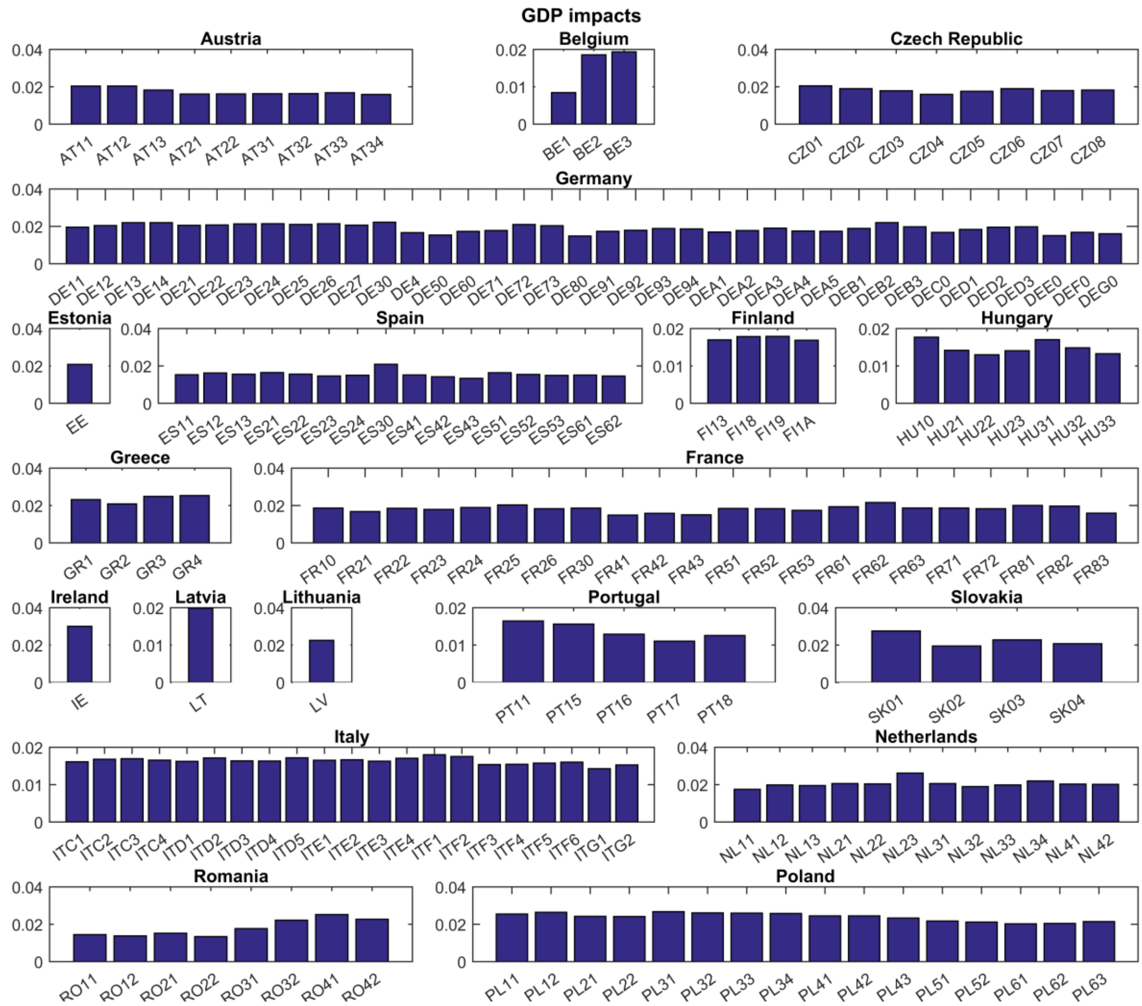

Figure 11 – Regional impacts of 1% REDI shocks on regional GDP levels

Conclusion

In this report we have developed a version of the GMR-Europe model which is capable of estimating the likely effects of policies which target entrepreneurship and/or network formation. As a consequence, this impact analysis tool is suitable for the evaluation of smart specialization policies which build on the regions' own traditions together with a combination of more traditional sector-neutral development policies and government-supported entrepreneurial discovery processes.

We have given a detailed account of the GMR-Europe policy impact model, which has been developed to facilitate impact assessment of smart specialization policies by specifically integrating variables describing the entrepreneurial ecosystem as well as the network embeddedness of European regions. The GMR models are structured around three model blocks. The Total Factor Productivity (TFP) block is able to capture the role of innovation-related factors such as R&D, human capital, entrepreneurship and knowledge networks in productivity growth at the regional level. A spatial computable general equilibrium (SCGE) block allows for the estimation of regional allocation and reallocation of resources as well as trade and migration as a result of given policy interventions. Finally, a macroeconomic (MACRO) model block generates the dynamics of key variables like employment, investment, capital stock. The complex interaction of these model blocks allows us to estimate the likely impacts of different innovation-oriented policy interventions both at the regional and aggregate levels in several dimensions (GDP, productivity, employment, etc.).

In addition to a detailed account of the model setup and estimation/calibration processes, we also reported a brief simulation exercise illustrating the potential capabilities of the model in evaluating entrepreneurship-related policies. In this simulation we have shown the estimated effects of a policy capable of improving the entrepreneurial climate/ecosystem of the regions in the model. This intervention is shown to have a positive effect on regional productivity levels, however the same relative improvement/intervention results in differing long run productivity impacts (even in relative terms) due to different regional characteristics in 'transferring' entrepreneurial development into productivity. Also, our results show how the dynamic interaction between regions through trade and mobility of production factors has additional cumulative effects on the economic output of regions. In some places the outflow of production resources yield a less favorable development path while others, able to attract these resources, show higher long run growth rates in economic output. Finally, although the improvement of the entrepreneurial climate is restricted to the first 5 years of the simulation period, there is a steady improvement in the productivity and hence the economic output of regions. This means that an initial push contributing to regional entrepreneurial activities is able to drive the regional economy on a long term development path through increasing productivity levels even after the policy ceases directly impacting entrepreneurship.

We also called the attention to the fact that the presented simulation being very aggregate, the detailed structure of the REDI index and the other factors involved in the GMR-Europe model allows for more sophisticated policy evaluation analyses in the future.

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APPENDICES

A.1 Calibration of TFP, GDP growth and substitution elasticity in the MACRO block

First, the calibrated TFP block is used to calculate the expected aggregate trend TFP for simulation scenarios. This is the aggregate TFP trend extrapolated into the future. As this consists of aggregating the regional TFP trends which provide the baseline run of the model, this extrapolated aggregate trend TFP can be regarded as a steady state for the MACRO block. Using these extrapolated aggregate TFP values, we calculated the average annual TFP growth which is then used as the steady state TFP growth rate in the MACRO block. This is 0.36% annually.

Second, we estimated the elasticity of regional TFP values to regional employment. To do this, we simulated the change in regional TFP as a consequence of a 1% increase in regional employment. This turns out to be 0.0676% on average (over the forecasting period). This value reflects the productivity effect of an increasing labor force in a given region, thus it is a measure of agglomeration economies. Assuming that this agglomeration economies is linked to increasing returns to scale, we match the size of this effect with the sum of exponents in the MACRO production function (equation M15). As the exponents of effective labor and private capital sums up to unity, increasing returns at the aggregate level is provided by the exponent of public capital, which is $(1-\alpha_G)$. Using this logic, we get 0.00068 for α_G.

Third, given the steady state growth rate of TFP (g^A), the elasticity of public capital $(1-\alpha_G)$, labor (α) and the steady state growth rate of intermediate technology (g^{AI}) we can use the production function (M15) to calculate the GDP growth rate consistent with these numbers. Using the steady state calculation of the model it can be shown that private and public capital stocks grow with the rate $\overline{GY}+g^{AI}$ in the steady state, capacity utilization and labor grow at rate 0. As a result, it follows from the production function that steady state GDP growth can be expressed as follows:

$$\overline{GY}=\frac{\alpha}{\alpha+\alpha_G-1}\cdot g^A+\frac{2-\alpha-\alpha_G}{\alpha+\alpha_G-1}\cdot g^{AI}$$

Using this formula, the steady state growth rate of GDP is 0.22% quarterly which corresponds to a 0.89% annual growth rate. This seems to be a moderate but not too pessimistic long run growth rate for the steady state.[15]

Fourth, the value of α_G is used to set the varieties elasticity parameter τ. The logic behind this is that in the MACRO block, α_G provides the increasing returns in the production function by raising the sum of exponents above unity. As it is proved in the description of the SCGE block, this is linked to the market power of firms which is derived from the finite substitution elasticities between product varieties. If this substitution elasticity is ϵ, the sum of powers in the production function must be $\epsilon/(\epsilon-1)$. As in the MACRO block the sum of powers in the production function exceeds unity exactly by α_G, it follows that

$$1+\alpha_G=\frac{\epsilon}{\epsilon-1}$$

It is straightforward to conclude that

$$\alpha_G=\frac{1}{\epsilon-1}$$

which states, that with ϵ being large the power of public capital reflects the reciprocal of the elasticity of substitution between product varieties, which is exactly parameter τ in the MACRO block.

A.2 Adjusting differing data structures in the GMR model blocks

As the three main blocks of the GMR model rely on partially different data structures, we need to apply some adjustments at several points to render the data consistent in the three blocks. These adjustments affect TFP values used in the different model blocks as this is the main matching point between the blocks.

Adjusting trends and reference year data

The main logic of the TFP block is that in the baseline it runs with trends of the variables in the two equations. As a result from trend fitting, the trend values of the TFP block variables are not the same as the original observed variables. On the other hand, the SCGE block is calibrated to the data of 2012 which means that employment and TFP values are not consistent in the base year of the SCGE block, which are the observed data, and the fitted trend values of the TFP block.[16]

In order to overcome this discrepancy, we shifted the trend lines of the TFP values in the TFP block. This amounts to calibrating regions-specific γ_r constants which ensure that in the base year 2012 the trend TFP values and the observed TFP values match for all regions. In every other year the trend TFP

[15] Note that we extrapolate tendencies to 20 years in these calculations, and also, if we used past growth rates of the sample period (between 1995 and 2016) the average annual growth rate wold be even lower, 0.66% annually.
[16] These two variables overlap between the two model blocks.

values are multiplied with the same regions-specific constant. The average of these adjustment constants is 3.5% relative to the TFP values in 2012.

Employment values between the two blocks are matched in a dynamic fashion. As changes in the employment levels in the TFP block come from the SCGE block, we simply use the relative changes in regional employment calculated by the SCGE block and update the employment data in the TFP block with these relative changes.

Adjusting trends and baseline calculations

In the baseline runs we want the TFP block to run along the fitted trends, i.e. that if the two equations of the TFP model are simulated with no shocks in an iterative manner, the resulting TFP values correspond to the fitted TFP trends for all regions. Due to the setup of the TFP block, however, this is not necessarily the case, which means that there is a difference between calculated regional TFP values from the TFP equations and the trend values even if the starting point is the same in the reference year.

This bias is solved with a similar logic as before in the base year. For every region and year we define a $\mu_{r,t}$ constant which is used to multiply calculated (simulated) TFP values in order to match the trend lines in every period. These constants are then used in the scenario runs as well to keep the consistency between baseline and scenario simulations. This way, running a baseline ensures that simulated regional TFP values run along their fitted trend values and also in 2012 they match the observed data. The average of these adjustment constants is 4.3% relative to the TFP values in 2012.

Adjusting aggregate TFP to the MACRO block steady state

A third point where adjustment is required is the level of the MACRO block. As described so far, in the baseline simulations regional TFP values correspond to their fitted trend values estimated from actual TFP data so that in the base year 2012 the trend value matches the observed data. As the fitted trend is linear and different for all regions, there is a different (decreasing) annual average (aggregate) TFP growth rate for every year in the baseline simulation. On the other hand, the setup of the MACRO block requires a constant steady state growth rate for the TFP.

In order to match the TFP/SCGE block baseline simulations (which provide a changing aggregate TFP growth rate over the simulation period) with the MACRO block baseline simulation (which is the steady state of the model requiring a constant TFP growth rate), we apply a third adjustment which amounts to calibrating τ_t time-specific constants. By multiplying the aggregate TFP levels calculated from the baseline simulations of the TFP/SCGE blocks with these constants we get TFP levels which are consistent with the constant growth rate assumption of the MACRO block. These constants are then used in the scenario simulations as well in order to keep the consistency between the baseline and scenario simulations.[17] The average of these adjustment constants is 0.06% relative to the TFP values throughout the simulation period.

A.3 Adjustments between the MACRO and SCGE model blocks during simulation

The adjustment of regional employment

The adjustment of each variable is done separately in the baseline and the scenario. In the baseline we use a region-neutral allocation while in the scenario we need to account for the impacts of policy interventions.

In case of baseline employment adjustment, we use the following method: first, we calculate the absolute difference between the macro (${Ts_L^{Ba}}_{t+1}$) and aggregated regional employment level:

$$dL^{Ba}_{Na\,t+1} = {Ts_L^{Ba}}_{t+1} - \sum_i L_{i,t}$$

We allocate the difference to regions based on their regional employment share in the previous time period:

$$dL^{Ba}_{Re\,i,t+1} = dL^{Ba}_{Na\,t+1} \cdot \frac{L_{i,t}}{\sum_i L_{i,t}}$$

$$L_{i,t+1} = L_{i,t} + dL^{Ba}_{Re\,i,t+1}$$

In the scenario we need to account for the impact of shocks on employment and we still need a neutral adjustment in those regions that are not affected by interventions. Thus, the baseline adjustment needs some further improvement. We separated the method into two steps. First, we reproduce the baseline value of employment thus we eliminate differences between the baseline and the scenario in case of regions that experience changes as a result of shocks. The original baseline adjustment would fail to fulfil this requirement since when the macro employment is changed it would allocate the difference to all regions not only to those that are affected by the shock. We calculate the difference between the baseline regional employment and the actual value of employment in the scenario to replicate the baseline employment:

$$dL^{Sc0}_{Re\;\,i,t+1} = L^{Ba}_{i,t+1} - L_{i,t}$$

$$L_{i,t+1} = L_{i,t} + dL^{Ba}_{Re\,i,t+1}$$

[17] Note that the steady state TFP growth rate of the MACRO block is derived from the fitted trend values by simulating the TFP block and calculating the average aggregate TFP growth rates from this simulation. It follows that these adjustments can not bee large.

In the second step we account for the additional change of macro employment:

$$dL^{Sc}_{Na\,t+1} = Ts^{Sc}_{L\ \ t+1} - \sum_i L_{i,t+1}$$

Then we redistribute this macro difference to regions based on the regional TFP change caused directly or indirectly by policy shocks weighted by employment level. Therefore the regional impact of a policy shock depends on the magnitude of the change in TFP augmented by agglomeration effects:

$$dL^{Sc}_{Re\,j,t+1} = dL^{Sc}_{Na\,t+1} \cdot \frac{\left(TFP^{Sc}_{j,t} - TFP^{Ba}_{j,t}\right) \cdot L_{j,t+1}}{\sum_j \left(TFP^{Sc}_{j,t} - TFP^{Ba}_{j,t}\right) \cdot L_{j,t+1}} \qquad \mathit{if}\ TFP^{Sc}_{j,t} > TFP^{Ba}_{j,t}$$

$$dL^{Sc}_{Re\,j,t+1} = 0 \qquad \mathit{if}\ TFP^{Sc}_{j,t} \leq TFP^{Ba}_{j,t}$$

Those regions that are not affected by shocks will not benefit from macro employment changes. In case of no or insignificant macro productivity change the adjustments above are replaced automatically by the baseline adjustment.

It is important to note that the first step eliminates impacts of changes of migration in the scenario since we reproduce exactly the baseline employment levels. Thus, in each time period after the adjustments above we account for the loss of cumulated migration (before time period t+1):

$$L_{i,t+1} = L_{i,t+1} + \sum_1^t L^{Mig}_{i,t}$$

Adjustment of regional investment

In the baseline, regional investment is calculated by allocating macro investment ($Ts^{Ba}_{Inv\,t}$) to regions based on their regional capital share:

$$Kinv^{Ba}_{i,t} = Ts^{Ba}_{Inv\,t} \cdot \frac{K_{i,t}}{\sum_i K_{i,t}}$$

In the scenario we account for the changes in macro investment:

$$dInv^{Sc}_{Na\,t} = Ts^{Sc}_{Inv\,t} - Ts^{Ba}_{Inv\,t}$$

Then we follow the same approach employed in case of regional employment and redistribute macro investment changes on the basis of employment weighted absolute TFP changes:

$$dInv^{Sc}_{Re\,j,t} = dInv^{Sc}_{Na\,t} \cdot \frac{\left(TFP^{Sc}_{j,t} - TFP^{Ba}_{j,t}\right) \cdot L_{j,t}}{\sum_j \left(TFP^{Sc}_{j,t} - TFP^{Ba}_{j,t}\right) \cdot L_{j,t}} \qquad \mathit{if}\ TFP^{Sc}_{j,t} > TFP^{Ba}_{j,t}$$

$$dInv^{Sc}_{Re\,j,t} = 0 \qquad \mathit{if}\ TFP^{Sc}_{j,t} \leq TFP^{Ba}_{j,t}$$

Then scenario investment is simply calculated as baseline investment updated by the redistribution:

$$Kinv_{i,t}^{Sc} = Kinv_{i,t}^{Ba} + dInv_{Re\,i,t}^{Sc}$$

Also, we must note that in case of a shock that does not cause significant macro TFP impacts the scenario adjustment of investment is carried out in the same manner as the baseline.

Furthermore, the adjusted investment in time period t will increase the capital stock in the next period according to the equation of capital accumulation:

$$K_{i,t+1} = (1-\delta) \cdot K_{i,t} + Kinv_{i,t}^{Sc}$$

Where δ is the depreciation rate. This capital stock will be further adjusted in the next step.

Adjustment of regional capital stock

Since the three components (value added, employment, capital stock) of the regional production function cannot be changed independently, one of them will be given by the value of the other two. In our approach we decided to prescribe prefect consistency between the macro and regional value of those variables that are known from data. Since the regional capital stock is estimated using PIM method we use this variable to ensure consistency in case of value added.

In the baseline we first calculate the relative difference ($Ts_{Sync_t}^{Ba}$) between macro ($Ts_{GVA_t}^{Ba}$) and aggregated regional GVA (calculated from by the regional Cobb-Douglas production function) in the SCGE block in each time period. In the first year this value is unity.

$$Ts_{Sync_t}^{Ba} = \frac{Ts_{GVA_t}^{Ba}}{\sum_i A_{i,t} \cdot L_{i,t}^{\alpha_i} \cdot K_{i,t}^{\beta_i}}$$

Then we adjust the level of regional capital stock in the regional production function in order to generate consistent regional GVA values:

$$K_{i,t} = K_{i,t} \cdot \left(Ts_{Sync_t}^{Ba}\right)^{\frac{1}{\beta_i}}$$

This means that we employ the same rate of adjustment in each region. However, the size of adjustment is influenced by β_i parameter which is different in each country but regions within the same country are characterized by the same value. Thus, the size of adjustment will be different in countries. It can be shown that by using the adjusted capital stocks the inconsistency of value added values is eliminated.

In the scenario we employ a slightly different approach. The adjustment of the capital stock has to be neutral too which means that the adjustment has to be independent of shocks at least in those regions where no intervention took place. The use of baseline adjustment in scenario would cause distortions similar to those we already mentioned in the case of employment adjustment. As a result, positive GVA changes could be found also in regions that are not affected by policy shocks and gain no positive spillover effects from those shocks. To overcome this problem, we separated the adjustment into two steps again. First, we adjust the capital stock of all regions according to the baseline method (neutral adjustment) then we further adjust capital stock of those regions that are affected by policy shocks to reach full consistency of GVA values.

First, we execute again the baseline adjustment using $Ts^{Ba}_{Sync_t}$. This does not mean that the baseline path of value added will be exactly equal to the scenario path in regions that are not affected directly by shocks. We do allow for migration and investment for example which can deviate those regional development paths from their baseline values but the relative size of adjustment is unchanged in this first step.

Second, we calculate the absolute difference between the scenario macro value added ($Ts^{Sc}_{GVA_t}$) and the aggregated regional value added calculated in the first step. This difference is approximately the effect of the policy interventions on value added thus this additional value added should be distributed in those regions that are directly affected by the shock (or by its spillovers). Thus, in order to calculate relative adjustment, we compare this absolute difference to the aggregated value added of those regions that are somehow affected positively by the shock in terms of their region TFP value.

$$Ts^{Sc}_{Sync_t} = 1 + \frac{Ts^{Sc}_{GVA_t} - \sum_i A_{i,t} \cdot L^{\alpha_i}_{i,t} \cdot K^{\beta_i}_{i,t}}{\sum_j A_{j,t} \cdot L^{\alpha_j}_{j,t} \cdot K^{\beta_j}_{j,t}} \qquad if\; TFP^{Sc}_{j,t} > TFP^{Ba}_{j,t}$$

Thus $Ts^{Sc}_{Sync_t}$ gives us the rate of adjustment of those regions that experienced TFP growth (compared to baseline) in scenario in order to ensure consistency between the macro and the regional level. The capital stock (and thus the value added) of those regions that experienced no or negative TFP change will be adjusted only in the first step. Thus, the adjustment of capital stock in scenario can be summarized as follows:

$$K_{j,t} = K_{j,t} \cdot \left(Ts^{Sc}_{Sync_t} \cdot Ts^{Sc}_{Sync_t}\right)^{\frac{1}{\beta_j}} \qquad if\; TFP^{Sc}_{j,t} > TFP^{Ba}_{j,t}$$

$$K_{i,t} = K_{i,t} \cdot \left(Ts^{Sc}_{Sync_t}\right)^{\frac{1}{\beta_i}} \qquad if\; TFP^{Sc}_{i,t} \leq TFP^{Ba}_{i,t}$$