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Comparative advantage in routine production *

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Abstract

We pin down a new mechanism behind comparative advantage by pointing out that countries differ in their ability to adjust to technological change. We take stock of the pattern extensively documented in the labor literature whereby more efficient machines displace workers from codifiable (routine) tasks. Our hypothesis is that labor reallocation across tasks is subject to frictions and that these frictions are country-specific. We incorporate task routineness into a canonical 2-by-2-by-2 Heckscher-Ohlin model. The key feature of our model is that factor endowments are determined by the equilibrium allocation of labor to routine and non routine tasks. Our model predicts that countries which facilitate labor reallocation across tasks become relatively abundant in non routine labor and specialize in goods that use non routine labor more intensively. We document that the ranking of countries with respect to the routine intensity of their exports is strongly connected to two institutional aspects: labor market institutions and behavioral norms in the workplace.

Keywords: comparative advantage, routineness

JEL codes: F11, F14, F15

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Executive Summary

Classic theories of international trade generated predictions on the comparative advantage of countries from differences in factor endowments (Heckscher-Ohlin) or directly from productivity differences (Ricardo). More recent contributions have postulated that comparative advantage might also be generated endogenously from cross-country differences in institutions. We propose a variation of that approach in order to predict the specialization of countries in goods that are intensive (or not) in labor input of routine-task.

We start from the production function pioneered by Acemoglu and Autor (2011) that models sectors as differing in their relative intensity of nonroutine labor input versus an intermediate input. Crucially, this intermediate input is produced itself using routine labor or machines which are imperfect substitution in a CES aggregator. Our innovation is to make the ease of substitutability in the CES function a dimension along which countries differ.

We first provide some evidence that our nested production function is able to fit some features of the data. We estimate the production function using the KLEMS dataset for 25 countries and 30 industries, exploiting only the time dimension and allowing the structural parameters to vary across both industry and country dimensions. We obtain the best fit of the data if we let the parameter of the outer nest, which captures the relative importance of nonroutine labor input, vary across sectors, while we let the parameter of the inner nest, which captures the easy of substitution between routine labor and capital in the production of the intermediate, vary across countries.

Next, we derive comparative advantage predictions from this production function and investigate which institutional characteristics of countries can support these predictions. We do this in a two-step procedure, borrowed from Costinot (2009). First, we estimate for each country how strong the correlation is between the sectoral composition of its net export bundle and the routine-intensity across sectors (a primitive of technology). Next, we investigate which institutional features are significant predictors of these correlation coefficients. The first step yields intuitive patterns: Countries that specialize the least in routine-intensive production are Japan, Switzerland, Germany, and Sweden, while countries that specialize the most in routine-intensive goods are Thailand, Italy, Canada, and China. It is interesting to note that these patterns are quite different from those obtained from traditional measures of skill-intensity. In the second step, we estimate that the institutional characteristics that co-vary positively with specialization in nonroutine-production are: rule of law, strong norm in the workforce (low absenteeism, flexibility, responsibility,...), and high internal migration.

Finally, we provide some avenues to start thinking how the production function that we proposed can be built up from micro-foundations. The function has been used widely, especially in labor economics, but it has generally been treated as an exogenous primitive of the economy. We describe a few mechanisms where the adjustment of an economy to an exogenous increase in labor-substituting capital is facilitated by flexible labor market institutions, e.g. low severance pay, the fraction of the cost of retraining workers that is borne by society (government) rather than individual firms, high quality of formal schooling that imparts general (not firm-specific) skills, etc. We illustrate how a primitive parameter measuring high severance pay, for example, leads to low substitutability between factors in routine intermediate production. These mechanisms suggest directly a number of mechanisms that governments can exploit to influence their comparative advantage away from routine-intensive production.

1 Introduction

The classical theory of comparative advantage puts forward that differences in technology and factor endowments lead countries to specialize in the production of different goods. Recent developments in this literature put forward the role of worker attributes (human capital, skill dispersion) and of institutions (the ability to enforce contractual relationships) in shaping the pattern of trade. The available evidence supports the view that countries differ in many dimensions, and that all of these dimensions play a role in determining the pattern of trade.[1]

We seek to contribute by merging the comparative advantage literature with a prominent topic in the labor literature. We pin down a new mechanism behind comparative advantage by noticing that countries may differ in their ability to adjust to technological change. Our starting point is a well-documented pattern associated to the recent process of technological change.[2] The operationality of more efficient machines leads to the displacement of workers away from the relatively more codifiable ('more routine') tasks in which the new machines have a comparative advantage. The automation of routine tasks frees up labor to perform the less codifiable ('non routine') tasks. We find that countries that are better able to reallocate workers across tasks specialize in goods that require more intensive use of labor in the non routine tasks.

To make this point we incorporate task routineness into an otherwise canonical 2-country 2-good 2-factor Heckscher-Ohlin model. The two factors needed for the production of the two final goods are the routine and the non-routine factor. The key feature of our model is that the available quantities of these two factors are not given exogenously. Instead, these quantities are determined by the equilibrium allocation of labor to routine and non-routine tasks. We model the process of technological change as an increase in the capital endowment. As in Autor et al. (2003) and Autor and Dorn (2013), we posit that capital can only be used in routine tasks. An increase in the quantity of capital brings about a reduction in the relative cost of capital and an increase in the equilibrium capital intensity of routine production. Consequently, labor can be released from routine tasks and reallocated to non-routine tasks.[3]

[1] Chor (2010) shows that institutional characteristics matter at least as much as factor endowments. Bombardini et al. (2012) show that the level of human capital and the degree of skill dispersion are quantitatively similar.

[2] See in particular Autor et al. (2003), Acemoglu and Autor (2011), Goos et al. (2014), Harrigan et al. (2016).

[3] Two approaches have been used to model labor reallocation. Autor and Dorn (2013) posit that workers can only be reallocated from routine tasks in manufacturing to manual non-routine tasks in services while Autor et al. (2003) allow reallocation from routine to non-routine tasks in manufacturing. We follow the approach of Autor et al. (2003) while relaxing their assumption of perfect capital-labor substitutability in routine tasks.

Our hypothesis is that the reallocation of workers across tasks is subject to frictions, and that the intensity of these frictions is country-specific. We model the intensity of frictions associated to the process of labor reallocation as a change in the elasticity of substitution between capital and labor in routine production. We think of this assumption as a reduced form approach to capturing differences in labor market regulations across countries as well as differences in worker bargaining power and, more generally, in the intensity of frictions in the workplace. Specifically, we expect the elasticity of substitution to be decreasing in the magnitude of hiring, firing, and retraining costs associated to the adjustment of the workforce to the new machines.

Our model delivers the prediction that countries which adjust more smoothly to technological change - i.e. countries with a higher elasticity of substitution between capital and labor in routine production - free up more labor for non-routine tasks and become non-routine labor abundant. As in the canonical Heckscher-Ohlin model, the abundance of non-routine labor makes these countries relatively more efficient in producing goods that use the non-routine labor more intensively. Consequently, we get the prediction that countries which adjust more smoothly to technological change specialize in goods that are relatively non-routine intensive. This new mechanism of comparative advantage helps to explain why countries with similar factor endowments and similar technology may specialize in different goods.

We test the predictions of our model by following the approach in Costinot (2009). We work with bilateral trade data at the HS 2-digit level in 2000-2006. To reduce the number of zeros in the trade matrix, we restrict the sample to the 19 biggest exporters and the 34 biggest importers. In the first step of the estimation, we rank countries with respect to the routine intensity of their exports.[4] In the second step, we regress the ranking of countries with respect to the routine intensity of their exports on their ranking with respect to institutional characteristics that likely correlate with the ability to reallocate labor across tasks.

We find that the strictness of employment protection legislation (EPL) helps to explain differences in specialization across the countries of the European Union. Consistently with the predictions of our model, European countries with relatively strict EPL - and hence, lower capital-labor substitutability - specialize in goods that are relatively routine-intensive. Further, we find that the quality of the workforce as well as behavioral norms in the workplace help to explain differences in specialization across the 19 biggest world exporters. Consistently with the predictions of our model, countries in which the labor force is more able and more reliable

[4] The ranking of industries with respect to routine intensity is taken from Autor et al. (2003). We match their ranking across 140 census industries to the HS 2-digit classification.

specialize in goods that are relatively non routine-intensive.

Our work connects to three strands of the literature. Starting from the seminal work by Klump and De La Grandville (2000) who showed that the magnitude of the elasticity of substitution between capital and labor had substantial implications for growth, macroeconomists have seeked to estimate the magnitude of capital-labor substitutability and to uncover its determinants. We contribute to this literature by connecting the magnitude of capital-labor substitutability to the institutional characteristics of countries and by showing that differences in capital-labor substitutability play a role in determining countries' specialization in trade.

Our work also connects to the rapidly growing literature in labor economics that documents how increased automation and outsourcing of codifiable tasks led to job polarization in developed economies. This literature explicitly connects technological change to labor displacement from routine to non-routine tasks.[5] We contribute to this literature by showing that institutional characteristics play a role in determining the cost of worker reallocation across tasks. Further, we document that workers are expected to benefit relatively more from trade in countries that are able to adjust more smoothly to technological change.

Last but not least, we contribute to the trade literature that seeks to uncover new mechanisms behind comparative advantage. As pointed out by Nunn and Trefler (2014), this literature has extensively documented the importance of institutional characteristics. Our work also underscores the role of institutions. Our main contribution consists in pointing out that institutions may have a direct effect on the adjustment of the economy to technological change and, consequently, on the allocation of labor across tasks. We show that differences in measured factor abundance such as country-specific ratios of the skilled to the unskilled labor may be determined by the interaction of institutional characteristics with the process of technological change.

The rest of this paper is organized as follows. In section 2 we present some evidence on the ranking of sectors with respect to the routine intensity of tasks and on the ranking of countries with respect to capital-labor substitutability in routine production. In section 3 we present the main features of our stylized model, derive the autarky equilibrium and discuss the predictions regarding the pattern of trade. In section 5 we discuss one possible microfoundation of differences in capital-labor substitutability. Specifically, we show that an increase in the magnitude of adjustment costs leads to a reduction in measured capital-labor substitutability. In section 4 we discuss the estimation strategy and our results. We conclude in section 6.

[5] See in particular Autor et al. (2003), Acemoglu and Autor (2011), Goos et al. (2014), Harrigan et al. (2016).

2 Stylized facts

Our starting point is the production function used by Autor et al. (2003) to study the impact of technological change on the evolution of employment in routine and non routine tasks. The authors assume Cobb-Douglas technology in each industry g whereby non-routine tasks A_g and routine tasks M_g are combined to obtain output Y_g.[6] The parameter $\beta_g \in (0,1)$ captures the intensity with which the industry uses the routine tasks.

$$Y_g = A_g^{1-\beta_g} M_g^{\beta_g}$$

Non-routine tasks are produced by non-routine labor L^a while routine tasks can be produced by both capital K and routine labor L^m. In Autor et al. (2003) capital and routine labor are perfect substitutes: $M_g = (L^m_g + K_g)$. In subsequent work capital and routine labor tend to be modelled as imperfect substitutes while maintaining the assumption of higher capital substitutability with routine than with non routine labor. In particular, Autor and Dorn (2013) posit $M_g = \left[(L^m_g)^\mu + (K_g)^\mu\right]^{1/\mu}$ where $\mu \in (0,1)$.

The assumption that industries can be ranked according to their routine-intensity (β_g) has become commonplace following the seminal work by Autor et al. (2003). Moreover, the mapping from the routine task intensity of occupations to the routine task intensity of industries proposed by Autor et al. (2003) has been widely used in empirical work. The magnitude of the elasticity $\sigma = (1-\mu)^{-1}$ that captures capital-labor substitutability in routine production has received less attention. In theoretical work this parameter is generally assumed to be common across industries and either equal or greater than 1.[7]

We retain the key feature of the production function in Autor et al. (2003) whereby capital and routine labor are relative substitutes while capital and non-routine labor are relative complements but we put forward that the elasticity of substitution may be country-specific. We get the following two-tier production function:

$$Y_{igt} = (L^a{}_{igt})^{1-\beta_g}\left[(L^m_{igt})^{\mu_i} + (K_{igt})^{\mu_i}\right]^{\frac{\beta_g}{\mu_i}} \tag{1}$$

We use the EU-KLEMS database that provides information on capital and labor use in 21 countries (i) and 30 industries (g) over 25 years (t) to substantiate this hypothesis.

[6] We modify notation in Autor et al. (2003) to be consistent with notation in the rest of this paper.

[7] In Goos et al. (2014) capital-labor substitutability in the production of each task is equal to 1 and common across industries. In Acemoglu and Restrepo (2016) capital and labor are perfect substitutes in routine tasks.

First, we carry out an analysis of variance regarding factor allocation to tasks. The EU-KLEMS database reports total employment in each industry together with its split across high-, medium-, and low-skilled occupations. In manufacturing, there is a strong negative correlation between the skill intensity and the routine intensity of occupations. We equate high-skill employment with labor use in non-routine tasks L^a_{igt} and the remaining employment with labor use in routine tasks L^m_{igt}.

In table 1 we report the fraction of variance in labor allocation to non-routine tasks attributable to the time, country, and industry dimensions of the data. If routine intensity is industry-specific, variation in non-routine employment will be mainly driven by the industry dimension of the data. If capital-labor substitutability is country-specific, the country dimension should also play a role in explaining variation in non-routine employment.[8]

Table 1: Intensity of labor use in non-routine tasks: $L^a_{igt}/(L^a_{igt}+L^m_{igt})$

Adjusted $R2$	**Level**	**Ln** $(\cdot)$
Year dummies (t)	0.035	0.063
Country dummies (i)	0.296	0.337
Industry dummies (g)	0.460	0.388

The industry dimension has the most explanatory power. It explains between 39 and 46% of the total variation. The time dimension has little explanatory power. It explains just 4-6% of the total variation. Together these results support the premise that routine intensity is a technological characteristic of the production process in the industry. We note that about 30-34% of the variance is attributable to the country dimension. This finding is consistent with our assumption that labor allocation to tasks is co-determined by country characteristics.

The EU-KLEMS database reports information on capital services used in each industry. This variable is reported as a time index which means that we cannot directly compare capital-routine labor ratios across countries or industries. Nevertheless, we can interact year dummies with country or industry dummies to assess whether changes in capital intensity of routine

[8] For any given relative wage, the capital intensity of routine production K_{igt}/L^m_{igt} is common across industries in country i and L^a_{igt}/L^m_{igt} is decreasing in β_g. But K_{igt}/L^m_{igt} is a function of capital-labor substitutability μ. In general, L^a_{igt}/L^m_{igt} will vary in the country dimension if capital-labor substitutability is country-specific.

production are better explained by the country or the industry dimension. Table 2 reports the results. The data strongly indicates greater explanatory power for the country dimension.

Table 2: Intensity of capital use in routine tasks: K_{igt}/L^{m}_{igt}

Adjusted $R2$	**All observations**	**Trimmed to 1-99 percentile**
Country-Year dummies ($i * t$)	0.661	0.628
Industry-Year dummies ($g * t$)	0.318	0.285

Second, we estimate the two-tier production function (1) in each country and industry on the full set of years.[9] We implement two sets of constraints. In the baseline approach, we constrain the parameters to be comprised between 0 and 1 while allowing each parameter to vary in the industry and country dimensions (β_{ig}, μ_{ig}). In the second approach, we further constrain routine intensity to be common across countries (β_g).

In table 3 we report summary statistics for the 662 estimated values of β_{ig} and μ_{ig} obtained with the benchmark approach. There is substantial variability in the estimated parameters. The median elasticity of substitution in routine production (σ_{ig}) is 1.75, but the interquartile range comprises near-unitary elasticities as well as perfect substitutability between capital and routine labor. The median routine intensity (β_{ig}) is 0.81, and the interquartile range is 0.57-0.98.

Table 3: Summary statistics for β and μ

Parameter estimate	**Median**	**Mean**	**Std. dev.**	**IQR**	**10-90%**	**5-95%**	**1-99%**
β_{ig}	0.81	0.74	0.26	0.57-0.98	0.37-1	0.16-1	0-1
μ_{ig}	0.43	0.53	0.45	0.06-1	0.03-1	0.01-1	0-1

In table 4 we investigate which dimension of the data (country or industry) is better able to explain variation in estimated μ_{ig}. The first column reports the share of explained variation for estimates of μ_{ig} obtained with the benchmark approach. The second column reports the share of

[9] Capital services K_{igt} are normalized to the median value of routine labor L^{m}_{igt} in each country and industry.

explained variation for estimates of μ_{ig} obtained when the routine intensity (β_g) is constrained to be an industry characteristic.

Table 4: Country and industry dimensions of variation in μ_{ig}

Adjusted $R2$	**Unconstrained:** β_{ig}	**Constrained:** β_g
Country and industry dummies	0.051	0.084
Country dummies	0.031	0.067
Industry dummies	0.018	0.014
Percent explained by country alone	65%	83%

Less than 10% of the total variance in the estimated μ_{ig} is attributable to the country or the industry dimensions. Nevertheless, the bulk of the explained variation is attributable to the country dimension. This result is particularly striking when we follow the approach of the labor literature and constrain the routine intensity β_g to be an industry-characteristic common to all countries. Our results lend support to the assumption common in the labor literature that capital-labor substitutability in routine production is common across industries. They also lend support to our hypothesis that capital-labor substitutability is country-specific.

3 The model

3.1 Autarky

The two countries are denoted $i \in \{1, 2\}$. Factor endowments of capital $\bar{K}$ and labor $\bar{L}$ are common to the two countries. The two final goods are denoted $g \in \{1, 2\}$. The production function for good g in country i is:

$$Y_{ig} = z_g (L^a_{ig})^{1-\beta_g} M_{ig}^{\beta_g} \tag{2}$$

where z_g is the technology parameter in production of good g; L^a_{ig} is the quantity of non-routine labor used in production of good g, M_{ig} is the quantity of the routine input used in production of good g, and β_g is the factor share of the routine input in production of good g. Throughout this section, we consider good 1 to be relatively non-routine: $\beta_1 < \beta_2$.

The Cobb-Douglas production function for each of the two final goods contains an inner component that describes how routine labor L_i^m and capital K_{ig} are combined to produce the routine input M_{ig}:

$$M_{ig} = A_i \left[\alpha_i K_{ig}^{\mu_i} + (1-\alpha_i)(L_{ig}^m)^{\mu_i} \right]^{1/\mu_i} \tag{3}$$

where A_i and α_i are (respectively) the efficiency and distribution parameters of the CES production function, and $\mu_i = (\sigma_i - 1)/\sigma_i$ captures the extent of capital-routine labor substitutability. These three parameters may be country-specific. Throughout this section, we consider country 1 to have higher substitutability between capital and routine labor: $\mu_1 > \mu_2$. Further, we assume that in both countries capital and routine labor are more substitutable than non-routine labor and the routine input: $0 < \mu_i < 1$.

The full expression of the production function is:

$$Y_{ig} = z_g (L_{ig}^a)^{1-\beta_g} \left\{ A_i \left[\alpha_i K_{ig}^{\mu_i} + (1-\alpha_i)(L_{ig}^m)^{\mu_i} \right]^{1/\mu_i} \right\}^{\beta_g} \tag{4}$$

We denote $M_i = \sum_g M_{ig}$ the total quantity of the routine input and $L_i^m = \sum_g L_{ig}^m$ the total quantity of labor allocated to its production. We denote $L_i^a = \bar{L} - L_i^m = \sum_g L_{ig}^a$ the total quantity of labor allocated to non routine tasks. The outer production function (2) replicates the canonical 2×2 Heckscher-Ohlin model with two goods and two factors. Hence, it is sufficient to establish that one of the two countries becomes relatively non-routine labor abundant to prove that under autarky this country produces relatively more output in the sector that uses non-routine labor more intensively: $L_i^a/M_i > L_{i'}^a/M_{i'} \Leftrightarrow Y_{i1}/Y_{i2} > Y_{i'1}/Y_{i'2}$. The objective of this section is to establish the conditions under which the high (low) -μ country becomes relatively non-routine labor abundant.

3.1.1 Production of the routine input (M_i)

We denote w_i the wage and r_i the cost of capital and posit that for any given relative wage w_i/r_i, routine labor and capital are combined in the same way in production of the two final goods.[10] Further, we note that capital can only be used to produce the routine input. The production function for the routine input M_i can thus be written:

$$M_i = A_i [\alpha_i \bar{K}^{\mu_i} + (1-\alpha_i)(L_i^m)^{\mu_i}]^{1/\mu_i} \tag{5}$$

[10] App.A.1 validates this assumption by establishing the uniqueness of the solution.

We use (5) to solve for the use of routine labor as a function of routine input production. Whenever $M_i \leq A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$, $L_i^m = 0$. Whenever $M_i > A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$:

$$L_i^m(M_i) = [1-\alpha_i]^{-\frac{1}{\mu_i}} \left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{1/\mu_i} \tag{6}$$

We denote by C_i^m the cost function for the routine input. For $M_i \in \left(0, A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}\right]$ the cost function is constant and equal to $r_i\bar{K}$. For $M_i > A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$ the cost function is:

$$C_i^m(M_i;w_i,r_i) = w_iL_i^m + r_i\bar{K} = w_i[1-\alpha_i]^{-\frac{1}{\mu_i}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{1/\mu_i} + r_i\bar{K} \tag{7}$$

Consequently, the marginal cost function is constant and equal to 0 for $M_i \leq A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$. When M_i exceeds $A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$, the marginal cost function is:

$$\{C_i^m(M_i;w_i,r_i)\}' = MC_i^m(M_i;\cdot) = w_i[1-\alpha_i]^{-\frac{1}{\mu_i}}A_i^{-\mu_i}M_i^{\mu_i-1}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} \tag{8}$$

The average cost function for $M_i \in \left(0, A_i\alpha_i^{1/\mu_i}\bar{K}\right]$ is given by $r_i\bar{K}/M_i$. It is decreasing from $+\infty$ to $r_iA_i^{-1}\alpha_i^{-1/\mu_i}$. One would always produce at least $M_i = A_i\alpha_i^{1/\mu_i}\bar{K}$ since the marginal cost is 0 in this range. We therefore focus on the segment that verifies $M_i > A_i\alpha_i^{1/\mu_i}\bar{K}$:

$$\frac{C_i^m(M_i;w_i,r_i)}{M_i} = AC_i^m(M_i;\cdot) = \frac{w_i}{M_i}[1-\alpha_i]^{-\frac{1}{\mu_i}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{1/\mu_i} + \frac{r_i\bar{K}}{M_i} \tag{9}$$

When the quantity produced approaches infinity, the average cost function approaches the marginal cost function:

$$\begin{aligned}\lim_{M_i\to\infty} AC_i^m(\cdot) &= \lim_{M_i\to\infty} \frac{w_i}{M_i}[1-\alpha_i]^{-\frac{1}{\mu_i}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{1/\mu_i} = \\ \lim_{M_i\to\infty} w_i[1-\alpha_i]^{-\frac{1}{\mu_i}}A_i^{-\mu_i}M_i^{\mu_i-1}&\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{(1-\mu_i)/\mu_i} = w_i[1-\alpha_i]^{-\frac{1}{\mu_i}}A_i^{-1}\end{aligned} \tag{10}$$

We check that the average cost function attains a minimum for a unique and finite value of M_i. The derivative of the average cost function is:

$$\frac{dAC_i^m(M_i;w_i,r_i)}{dM_i} = \frac{w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\bar{K}^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} - r_i\bar{K}}{M_i^2} \tag{11}$$

The derivative is non-negative when:

$$w_i \; (1-\alpha_i)^{-\frac{1}{\mu_i}}$$

$$\alpha_i\bar{K}^{\mu_i} \left[\frac{M_i}{\frac{\mu_i}{A_i}} - \alpha_i\bar{K}^{\mu_i} \right]^{\frac{(1-\mu_i)}{\mu_i}} - r_i\bar{K} \geq 0 \tag{12}$$

Rearranging and simplifying gives:

$$\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \geq \left(\frac{w_i}{r_i}\right)^{\frac{-\mu_i}{1-\mu_i}} \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\mu_i}} \alpha_i \bar{K}^{\mu_i} \tag{13}$$

We solve for M_i to show that the average cost function is increasing whenever:

$$M_i \geq A_i \alpha_i^{\frac{1}{\mu_i}} \bar{K} \left\{ 1 + \left(\frac{w_i}{r_i}\right)^{\frac{-\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} \right\}^{\frac{1}{\mu_i}} \tag{14}$$

The term in the curly brackets is strictly bigger than 1. Consequently, the average cost function is decreasing between $A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$ and $A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}\left(1+[w_i/r_i]^{\frac{-\mu_i}{1-\mu_i}}[(1-\alpha_i)/\alpha_i]^{\frac{1}{1-\mu_i}}\right)^{1/\mu_i}$ and increasing thereafter. Thus, the average cost of producing the routine input is minimized at:

$$M_i^* = A_i \alpha_i^{\frac{1}{\mu_i}} \bar{K} \left\{ 1 + \left(\frac{w_i}{r_i}\right)^{\frac{-\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} \right\}^{\frac{1}{\mu_i}} \tag{15}$$

This result indicates that for any set of finite factor prices the optimal choice of routine input production M_i^* is strictly bigger than $A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$. It follows that some labor will be allocated to the production of the routine input in each country as long as μ_i is bounded away from 1.

Plugging (15) into (6) delivers the optimal quantity of routine labor:[11]

$$L_i^{m*} = \left(\frac{w_i}{r_i}\right)^{-\frac{1}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} \bar{K} \tag{16}$$

Using labor market clearing together with the condition that ensures optimal routine input production (15), we obtain the optimal ratio of non routine labor to the routine input for country i derived from the solution of the inner problem:

$$\frac{L_i^a(M_i^*)}{M_i^*} = \frac{\bar{L} - L_i^m(M_i^*)}{M_i^*} = \frac{\bar{L} - \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{-\frac{1}{1-\mu_i}} \bar{K}}{A_i \alpha_i^{\frac{1}{\mu_i}} \bar{K} \left(1 + \frac{w_i}{r_i}\left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{-\frac{1}{1-\mu_i}}\right)^{\frac{1}{\mu_i}}} \tag{17}$$

The solution to the inner problem also delivers the price of the routine input. Plugging the optimal choice of labor allocation to routine tasks (16) in the cost function (7) gives:

$$C_i^m(M_i^*; w_i, r_i) = r_i \bar{K} \left(1 + \frac{w_i}{r_i}\left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{-\frac{1}{1-\mu_i}}\right) \tag{18}$$

[11] This quantity corresponds to the relative factor demand that would be chosen to produce M_i^* if both capital and routine labor were freely chosen.

Consequently, the price of the routine input is:

$$P_i^m(M_i^*;\cdot) = \frac{C_i^m(M_i^*)}{M_i^*} = \frac{r_i\alpha_i^{-\frac{1}{\mu_i}}}{A_i}\left[1+\frac{w_i}{r_i}\left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{-\frac{1}{1-\mu_i}}\right]^{\frac{\mu_i-1}{\mu_i}} \tag{19}$$

We rearrange (19) to demonstrate that the price of the routine input mimicks the CES price index that would be obtained if both production factors were freely chosen:

$$\begin{aligned} P_i^m(M_i^*;\cdot) &= A_i^{-1}\alpha_i^{\frac{\mu_i-1}{\mu_i}}\frac{r_i}{\alpha_i}\left[\left(\frac{r_i/\alpha_i}{r_i/\alpha_i}\right)^{\frac{\mu_i}{\mu_i-1}} + \frac{1-\alpha_i}{\alpha_i}\left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{\frac{\mu_i}{\mu_i-1}}\right]^{\frac{\mu_i-1}{\mu_i}} = \\ & A_i^{-1}\alpha_i^{\frac{\mu_i-1}{\mu_i}}\left[\left(\frac{r_i}{\alpha_i}\right)^{\frac{\mu_i}{\mu_i-1}} + \frac{1-\alpha_i}{\alpha_i}\left(\frac{w_i}{1-\alpha_i}\right)^{\frac{\mu_i}{\mu_i-1}}\right]^{\frac{\mu_i-1}{\mu_i}} = \\ A_i^{-1}\left[\alpha_i\left(\frac{r_i}{\alpha_i}\right)^{\frac{\mu_i}{\mu_i-1}} + (1-\alpha_i)\left(\frac{w_i}{1-\alpha_i}\right)^{\frac{\mu_i}{\mu_i-1}}\right]^{\frac{\mu_i-1}{\mu_i}} &= A_i^{-1}\left[\alpha_i^{\sigma_i}r_i^{1-\sigma_i} + (1-\alpha_i)^{\sigma_i}w_i^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}} \end{aligned} \tag{20}$$

Denote the effective relative cost of labor by $\varpi_i = [w_i/(1-\alpha_i)]/[r_i/\alpha_i]$. From (16), the elasticity of substitution between capital and routine labor at the cost-minimizing choice of routine input production is $\sigma_i = d\ln[\bar{K}/L_i^{m*}]/d\ln(\varpi) = (1-\mu_i)^{-1}$. If the effective cost of labor were common to the two countries, the production of the routine input would be relatively labor-intensive in the high-μ country whenever the effective cost of labor is relatively low ($\varpi <$ 1). Further, from the general mean property of the CES production function we know that the quantity of the routine output M_i is strictly increasing in μ whenever the two countries allocate the same combination of inputs $\{K, L^m\}$ to its production and $K \neq L^m$ (Klump et al. (2012)).[12] Whenever labor is relatively cheap $\varpi < 1$, we know from (16) that the high-μ country allocates relatively more labor to routine tasks $L_1^{m*} > L_2^{m*}$ and from the general mean property that $M_1(L_2^{m*}) > M_2(L_2^{m*})$. It follows that the high-μ country must be relatively routine abundant: $L_1^a/M_1 < L_2^a/M_2$.

Whenever labor is relatively expensive $\varpi > 1$, we know from (16) that the high-μ country allocates relatively less labor to routine tasks $L_1^{m*} < L_2^{m*}$ whereby $L_1^{a*} > L_2^{a*}$. However, this is insufficient to prove that the high-μ country is relatively non routine abundant because from the general mean property we only know that $M_1(L_1^{m*}) > M_2(L_1^{m*})$. If we compute the derivative

[12] In the very special case of $K = L^m$ ($\varpi = 1$) the two countries obtain the same amount of the routine input.

of M_i^* with respect to μ while keeping ϖ fixed, we get:

$$\frac{\partial \ln M_i^*}{\partial \mu} = \frac{d}{d\mu}\left\{\frac{1}{\mu}\ln\left[\alpha_i\left(1+\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\right)\right]\right\} = -\mu^{-2}(1-\mu)^{-2}\left(1+\frac{w_i}{r_i}\varpi_i^{-\frac{1}{1-\mu}}\right)^{-1}\left\{(1-\mu)^2\left(1+\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\right)\ln\left[\alpha_i\left(1+\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\right)\right]+\mu\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\ln\varpi\right\}$$

We can show that $\alpha_i\left(1+\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\right) < 1$ iff $\varpi_i > 1$. Consequently, the first term in the curly brackets is strictly negative while the second term is strictly positive whenever $\varpi > 1$. It is sufficient to prove that the expression in curly brackets is non-negative to establish that the high-μ country is non-routine labor abundant: $L_1^a/M_1 > L_2^a/M_2$ whenever $\varpi > 1$ (given $M_1(L_1^{m*}) \leq M_2(L_2^{m*})$. However, the sign of the expression in curly brackets is ambiguous.[13]

It is important to realize that studying the partial derivative of routine input production with respect to μ does not inform us on the pattern of specialization because, as we show in the next section, the equilibrium wage is itself a function of μ.[14] Further, the pattern of specialization is established through the ratio $(L_1^a/M_1)^*/(L_2^a/M_2)^*$ rather than through the ratio M_1^*/M_2^*. Specifically, to prove that the high-μ country is non-routine abundant whenever labor is scarce, we need to establish that $L_1^{a*}/L_2^{a*} > M_1^*/M_2^*$.

The inner problem does not suffice to pin down the factor price ratio. Hence, we consider the outer problem to obtain the second expression of the optimal ratio of non routine labor to the routine input.

3.1.2 Production of final output

The outer problem is standard Heckscher-Ohlin. Costs are minimized in production of final good g by choosing M_{ig} and L_{ig}^a subject to the technological constraint $Y_{ig} \leq z_g \left(L_{ig}^a\right)^{1-\beta_g} M_{ig}^{\beta_g}$ taking factor prices P_i^m and w_i as given. The solution to this problem delivers relative factor demand:

$$\frac{L_{ig}^a}{M_{ig}} = \frac{1-\beta_g}{\beta_g}\frac{P_i^m}{w_i} \tag{21}$$

Denote Q_{ig} the consumption and P_{ig} the price of each final good. Goods' market clearing $Q_{ig} = Y_{ig}$ delivers $Q_{ig} = z_g \left(L_{ig}^a\right)^{1-\beta_g} M_{ig}^{\beta_g}$. Using (21) to substitute for M_{ig} defines the

---

[13] We can prove that the expression in curly brackets is negative whenever $\varpi \leq (1-\alpha_i)^{\frac{-(1-\mu)^2}{\mu(2-\mu)}}$. Further, the expression is likely to be negative even if the above inequality does not hold.

[14] The equilibrium factor prices must verify the first order conditions for the inner CES and the outer Cobb-Douglas production function in each country. The factor price ratio is country-specific whenever $\varpi_i \neq 1$.

consumption of each good as a function of non-routine labor used in production:

$$Q_{ig} = z_g L^a_{ig} \left(\frac{\beta_g}{1-\beta_g}\right)^{\beta_g} \left(\frac{w_i}{P^m_i}\right)^{\beta_g} \tag{22}$$

Using the zero profit condition $P_{ig} z_g \left(L^a_{ig}\right)^{1-\beta_g} M_{ig}^{\beta_g} = w_i L^a_{ig} + P^m_i M_{ig}$ together with (21) to substitute for $P^m_i M_{ig}$ allows solving for the price of each final good:

$$P_{ig} = \frac{w_i^{1-\beta_g} (P^m_i)^{\beta_g}}{z_g (\beta_g)^{\beta_g} (1-\beta_g)^{(1-\beta_g)}} \tag{23}$$

We get the second expression of the ratio of non-routine labor to routine input in each country by considering the consumer problem. We take a standard Cobb-Douglas utility function for the two final goods: $U_i = \sum_g \theta_g \ln(Q_{ig})$. The budget constraint is $\sum_g P_{ig} Q_{ig} \leq r_i \bar{K} + w_i \bar{L}$. The solution to the consumer problem gives an expression of total expenditure on one good as a function of relative income shares of each good and expenditure on the other good:

$$P_{i2} Q_{i2} = \frac{\theta_2}{\theta_1} P_{i1} Q_{i1} \tag{24}$$

Plugging (22) and (23) into (24) gives non-routine labor allocation to one sector as a function of labor allocation to the other sector:

$$L^a_{i2} = \frac{\theta_2}{\theta_1} \frac{1-\beta_2}{1-\beta_1} L^a_{i1} \tag{25}$$

Using (25) together with $L^a_i = L^a_{i1} + L^a_{i2}$ allows expressing non-routine labor use in sector 1 as a function of total non-routine labor use:

$$L^a_{i1} = L^a_i \frac{\theta_1(1-\beta_1)}{\theta_1(1-\beta_1) + \theta_2(1-\beta_2)} \tag{26}$$

Using (21) to substitute for L^a_i in (22) and plugging the resulting expression for Q_{ig} in (24) delivers the equivalent of (26) for the use of routine input in sector 1 as a function of total routine input use:

$$M_{i1} = M_i \frac{\theta_1 \beta_1}{\theta_1 \beta_1 + \theta_2 \beta_2} \tag{27}$$

Using (21) together with (26) and (27) gives an expression for the optimal relative use of non-routine labor and the routine input as a function of the factor price ratio:

$$\frac{L^{a*}_i}{M_i} = \frac{\sum_g \theta_g (1-\beta_g)}{\sum_g \theta_g \beta_g} \cdot \frac{P^m_i}{w_i} \tag{28}$$

We denote $c = \frac{\sum_g \theta_g (1-\beta_g)}{\sum_g \theta_g \beta_g}$ and replace the price of the routine input by its value in (19) to get:

$$\frac{L_i^{a*}}{M_i^*} = c \left[\frac{w_i}{r_i} A_i \alpha_i^{\frac{1}{\mu_i}} \right]^{-1} \left[1 + \frac{w_i}{r_i} \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right]^{\frac{\mu_i - 1}{\mu_i}} \tag{29}$$

In the next section we pin down the equilibrium factor price ratio by combining the expression of optimal factor allocation in the production of the routine input with optimal factor allocation in the production of the two final goods. We thereafter evaluate the ratio $(L_1^{a*}/M_1^*)/(L_2^{a*}/M_2^*)$ as a function of μ.

3.1.3 Equilibrium factor price ratio

The solution to the inner and outer problems each deliver an expression for the relative use of non-routine labor and of the routine input in final good production as a function of the factor price ratio and of capital-routine labor substitutability. We solve for the equilibrium factor price ratio by equating (29) with (17):

$$\left(\frac{w_i}{r_i} \right)^{-1} c \left[1 + \frac{w_i}{r_i} \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right]^{\frac{\mu_i - 1}{\mu_i}} = \frac{\frac{\bar{L}}{\bar{K}} - \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}}}{\left[1 + \frac{w_i}{r_i} \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right]^{\frac{1}{\mu_i}}}$$

Rearranging and simplifying gives:

$$\left(\frac{w_i}{r_i} \right)^{-1} c \left[1 + \frac{w_i}{r_i} \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right] = \frac{\bar{L}}{\bar{K}} - \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}}$$

$$\left(\frac{w_i}{r_i} \right)^{-1} c + c \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} = \frac{\bar{L}}{\bar{K}} - \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}}$$

We obtain an implicit solution for the equilibrium factor price ratio $\omega^* = (w_i/r_i)^*$:

$$\omega_i^* = c \left[\frac{\bar{L}}{\bar{K}} - (1+c) \left(\frac{1-\alpha_i}{\alpha_i} \right)^{\frac{1}{1-\mu_i}} (\omega_i^*)^{\frac{-1}{1-\mu_i}} \right]^{-1} \tag{30}$$

To establish existence and uniqueness of the solution, we define $F_i(\cdot)$:

$$F_i\left(\omega^*; \mu_i, c, \alpha_i, A_i, \frac{\bar{L}}{\bar{K}} \right) = (\omega^*)^{-1} c + (1 + c)$$

$$(\omega^*)^{-\frac{1}{1-\mu_i}} \quad 1- \frac{1-\alpha_i}{\alpha_i} = 0 \qquad (31)$$

$-\bar{K}$ i $\frac{1}{}$

Without loss of generality, we focus on cases where σ_i is an integer. We eliminate negative exponents by factoring out $(\omega_i^*)^{-\frac{1}{1-\mu_i}}$ and use $\sigma_i = (1-\mu_i)^{-1}$ and $\sigma_i - 1 = \mu_i/(1-\mu_i)$ to show that the solution is the root of the polynomial of degree σ_i:

$$
\begin{aligned}
F_i\left(\omega_i;\mu_i,\frac{\bar{L}}{\bar{K}},c,\alpha_i,A_i\right) &= -\frac{\bar{L}}{\bar{K}}(\omega_i^*)^{\frac{1}{1-\mu_i}} + c(\omega_i^*)^{\frac{\mu_i}{1-\mu_i}} + (1+c)\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} = 0 \\
\Leftrightarrow \frac{\bar{L}}{\bar{K}}(\omega_i^*)^{\sigma_i} - c(\omega_i^*)^{\sigma_i-1} - (1+c)\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\sigma_i} &= 0
\end{aligned}
\tag{32}
$$

The derivative with respect to ω_i^* is:

$$
\frac{\partial F(\cdot)}{\partial \omega_i^*} = -\sigma_i(\omega_i^*)^{\sigma_i-1}\frac{\bar{L}}{\bar{K}} + c(\sigma_i-1)(\omega_i^*)^{\sigma_i-2} = -\sigma_i(\omega_i^*)^{\sigma_i-1}\left[\frac{\bar{L}}{\bar{K}} - c(\omega_i^*)^{-1}\right] - c(\omega_i^*)^{\sigma_i-2}
$$

A sufficient condition for this derivative to be negative is to verify $\left[\bar{L}/\bar{K} - c(\omega_i^*)^{-1}\right] \geq 0$ or, equivalently, $\omega_i^* \geq c\bar{K}/\bar{L}$. By assumption, $\sigma_i \in (1, \infty)$. The function $F(\cdot)$ is monotonically decreasing in ω_i^*, it is positive for $\omega_i^* \to 0$ and negative for $\omega_i^* \to \infty$. We conclude that whenever $\omega_i^* \geq c\bar{K}/\bar{L}$, there exists a positive solution, and it gives rise to a finite real root ω_i^* that is the unique solution of this polynomial in each country.

The degree of the polynomial is country specific, and the solution to any polynomial in terms of its coefficients is degree-specific. Nevertheless, given the uniqueness of the solution, we can always express the solution of the polynomial in country 1 as a function of the solution in country 2: $\omega_1^* = \omega_2^*/v$.

3.1.4 Quantities and prices in autarky

We now characterize the pattern of production in autarky. From the consumer problem:

$$
\frac{Q_{i1}}{Q_{i2}} = \rho\left(\frac{L_i}{M_i}\right)^{\beta_2-\beta_1}
\tag{33}
$$

where $\rho = (z_1\theta_1)/(z_2\theta_2)c^{\beta_1-\beta_2}(1-\beta_1)^{1-\beta_1}\beta_1^{\beta_1}(1-\beta_2)^{\beta_2-1}\beta_2^{-\beta_2}$.

Since ρ is invariant across countries and $\beta_2 > \beta_1$ by assumption, it is sufficient to show that one country is relatively non-routine labor abundant to prove that under autarky this country will have relatively high consumption of the good that uses non-routine labor more intensively. Equivalently, we can investigate in which country the relative price of the routine input $(P_i^m/w_i)^*$ is higher in equilibrium (28).

We use (19) to write the relative price of the routine input as a function of parameters and of the equilibrium factor price ratio:

$$\frac{P_i^{*m}}{w_i} = \left(\frac{w_i}{r_i}\right)^{*} \left(A_i \alpha_i^{\frac{\mu_i}{\mu_i-1}}\right)^{-1} \left[1+\left(\frac{w_i}{r_i}\right)^{*\frac{\mu}{1-\mu}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu}}\right]^{\frac{\mu_i-1}{\mu_i}} \tag{34}$$

Next, we write the ratio of the relative price in the two countries as a function $G(v)$:

$$\frac{(P_1^m/w_1)^*}{(P_2^m/w_2)^*} = G(v) = \frac{\left[A_1\alpha_1^{\frac{1}{\mu_1}}(\omega_2^*/v)\right]^{-1}\left[1+(\omega_2^*/v)^{-\frac{\mu_1}{1-\mu_1}}\left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{1-\mu_1}}\right]^{\frac{\mu_1-1}{\mu_1}}}{\left[A_2\alpha_2^{\frac{1}{\mu_2}}\omega_2^*\right]^{-1}\left[1+(\omega_2^*)^{-\frac{\mu_2}{1-\mu_2}}\left(\frac{1-\alpha_2}{\alpha_2}\right)^{\frac{1}{1-\mu_2}}\right]^{\frac{\mu_2-1}{\mu_2}}} \tag{35}$$

We denote $C = \left(A_2\alpha_2^{\frac{1}{\mu_2}}/A_1\alpha_1^{\frac{1}{\mu_1}}\right)\left[1+(\omega_2^*)^{-\frac{\mu_2}{1-\mu_2}}\left(\frac{1-\alpha_2}{\alpha_2}\right)^{\frac{1}{1-\mu_2}}\right]^{\frac{1-\mu_2}{\mu_2}}$ and take the derivative of the function with respect to v to get:

$$\frac{dG(v)}{dv} = C\left[1+(\omega_2^*/v)^{-\frac{\mu_1}{1-\mu_1}}\left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{1-\mu_1}}\right]^{\frac{\mu_1-1}{\mu_1}}\left\{1-\frac{(\omega_2^*/v)^{-\frac{\mu_1}{1-\mu_1}}\left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{1-\mu_1}}}{1+\left(\omega_2^*/v\right)^{-\frac{\mu_1}{1-\mu_1}}\left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{1-\mu_1}}}\right\}$$

The expression in curly brackets captures the two effects that v has on the relative price of the routine input in the two countries. The positive effect works through the wage: when v increases, the wage in the high-μ country decreases relatively to the wage in the low-μ country whereby the routine input becomes relatively more expensive in the high-μ country. The negative effect works through the price index: the price of the routine input is reduced when labor becomes cheaper. The positive effect always dominates: the derivative is always positive. Consequently, if we can characterize v as a function of endowments and parameters and identify the value of v at which the ratio $(P_1^m/w_1)/(P_2^m/w_2) = 1$, we can establish the range of endowments for which the high-μ country is non routine labor abundant.

We can learn more about the magnitude of v by computing the partial derivative of the equilibrium factor price ratio with respect to μ. We apply the implicit function theorem to $F_i(\cdot)$ in (31) whereby:

$$\frac{\partial (w_i/r_i)^*}{\partial \mu} = -\frac{\partial F_i(\cdot)/\partial \mu}{\partial F_i(\cdot)/\partial (w_i/r_i)^*} \tag{36}$$

The partial derivative of $F_i(\cdot)$ with respect to the factor price ratio is negative:

$$\frac{\partial F_i(\cdot)}{\partial (w_i/r_i)^*} = -\left[c+(1+c)\left(\frac{w_i}{r_i}\right)^{*\,-\frac{\mu}{1-\mu}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu}}\right]^{-2} < 0 \tag{37}$$

$\partial (w_i/r_i)^* \quad - \quad r_i \quad 1-\mu \quad r_i \quad \alpha_i$

It follows that the sign of $\partial(w_i/r_i)^*/\partial\mu$ is determined by the sign of $\partial F_i(\cdot)/\partial\mu$. Recall that the effective cost of labor is $\varpi_i = [w_i/(1-\alpha_i)]/[r_i/\alpha_i]$. We get:

$$\frac{\partial F_i(\cdot)}{\partial \mu} = -\frac{(1+c)}{(1-\mu)^2}\varpi_i^{-\frac{1}{1-\mu}}\ln\varpi_i \tag{38}$$

We learn that labor is relatively cheap in the high-μ country when the effective cost of labor is high ($\varpi_i > 1$). Further, labor is relatively expensive in the high-μ country when the effective cost of labor is relatively low ($\varpi_i < 1$):

$$\begin{cases} \frac{\partial (w_i/r_i)^*}{\partial \mu} < 0, & \varpi_i > 1 \Leftrightarrow \frac{dv}{d\mu} > 0 \\ \frac{\partial (w_i/r_i)^*}{\partial \mu} = 0, & \varpi_i = 1 \Leftrightarrow \frac{dv}{d\mu} = 0 \\ \frac{\partial (w_i/r_i)^*}{\partial \mu} > 0, & \varpi_i < 1 \Leftrightarrow \frac{dv}{d\mu} < 0 \end{cases}$$

Further, we compute the derivative of the relative price with respect to the relative wage:

$$\frac{d(P_i^m/w_i)^*}{d(w_i/r_i)^*} = -\alpha_i \left[1 + \left(\frac{w_i}{r_i}\right)^{-\frac{\mu}{1-\mu}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu}}\right]^{-\frac{1}{\mu}} A^{-1} \left(\frac{w_i}{r_i}\right)^{-2} < 0 \tag{39}$$

Combining this derivative with our previous result on the effect of μ on the equilibrium relative wage delivers the result that the relative price of the routine input is increasing in μ whenever labor is relatively expensive.

$$\begin{cases} \frac{d(P_i^m/w_i)^*}{d(w_i/r_i)^*}\frac{\partial (w_i/r_i)^*}{\partial \mu} < 0 & \varpi_i < 1 \\ \frac{d(P_i^m/w_i)^*}{d(w_i/r_i)^*}\frac{\partial (w_i/r_i)^*}{\partial \mu} = 0 & \varpi_i = 1 \\ \frac{d(P_i^m/w_i)^*}{d(w_i/r_i)^*}\frac{\partial (w_i/r_i)^*}{\partial \mu} > 0 & \varpi_i > 1 \end{cases}$$

We learn that the factor cost channel pushes the high-μ country to specialize in non-routine production whenever labor is expensive and to specialize in routine production whenever labor is cheap. When labor is expensive, the routine input is relatively expensive in the high-μ country because labor in this country is relatively cheap, and the direct effect of the wage on the relative price of the routine input exceeds the indirect effect through which lower labor cost reduces the price of the routine input. It remains to be shown that $(P_1^m/w_1)^* = (P_2^m/w_2)^*$ when $\varpi_i = 1$ to prove that the low-μ country is non-routine labor abundant while $\varpi_i < 1$ and that the high-μ country becomes non-routine labor abundant when $\varpi_i > 1$.

3.1.5 Normalization of the CES production function

Klump et al. (2012) explain the rationale behind the normalization of the CES production function. Here we briefly summarize their argument. The CES is defined as the production function that possesses the following property: $\sigma = d\ln(K/L)/d\ln(F_k/F_l)$ is constant. This definition can be re-written as a second-order differential equation of $F(K, L)$. When one solves this second-order differential equation for F, one introduces 2 integration constants which are fixed by some boundary conditions.

The key point is that the elasticity of substitution is implicitly defined as a point elasticity, i.e. it is related to a particular point on a particular isoquant. But if the isoquant has to go through one particular point, the choices of the integration constants will depend on σ. Hence, the elasticity of substitution is the only structural parameter of the production structure. The properties of the boundary conditions, e.g. the capital share at the benchmark, will also influence the other parameters (together with σ).

Comparative statics in σ that do not adjust the integration constants compare situations where the isoquants for the initial and the final CES cannot be tangent at the benchmark point while the definition of σ requires that they have the same factor proportions and the same marginal rate of technical substitution at the benchmark point. One could take the full derivative of σ, incorporating the change in the other parameters explicitly. Alternatively, one can normalize the CES by making it go through an initial point (with Y_0, K_0, L_0 and a capital share π_0) and thus getting rid of all parameters other than σ. The normalization allows focusing on the structural effect of higher substitutability, e.g. the reduced incidence of decreasing marginal factor products.[15]

We pin down the relationships between ϖ_i, ν, and factor abundance in the two countries by normalizing the CES production function. The normalization point is defined by the level of routine production $\tilde{M}$, the capital-routine labor ratio $\tilde{\kappa} = \tilde{K}/\tilde{L}^m$ and the marginal rate of substitution $\tilde{\omega} = \tilde{w}_i/\tilde{r}_i = [(1-\alpha_i)/\alpha_i]\,\tilde{\kappa}^{1-\mu_i}$ such that at this point the capital and labor allocation to routine input production is independent of the substitutability parameter μ (Klump et al. (2012); Klump and De La Grandville (2000)).

The normalized coefficient on capital α_i is:

$$\alpha_i(\mu) = \frac{\tilde{\kappa}^{1-\mu}}{\tilde{\kappa}^{1-\mu} + \tilde{\omega}} \tag{40}$$

Routine input production at the point of normalization is used to define the normalized productivity term A_i:

$$\tilde{M} = A_i(\mu)\left[\alpha_i(\mu)\left(\tilde{K}\right)^{\mu} + [1-\alpha_i(\mu)]\left(\tilde{L}^m\right)^{\mu}\right]^{1/\mu} \Leftrightarrow$$

$$A_i(\mu) = \frac{\tilde{M}}{\tilde{L}^m}\left[\frac{\tilde{\kappa}^{1-\mu} + \tilde{\omega}}{\tilde{\kappa} + \tilde{\omega}}\right]^{1/\mu} \tag{41}$$

We now reformulate key relationships in terms of deviation from the point of normalization.

[15] σ is decreasing in the cross-partial derivative of production with respect to capital and labor.

Denoting optimal factor allocation in routine input production by $\kappa_i^* = \bar{K}/L_i^{m*}$, (16) becomes:

$$\frac{\kappa_i^*}{\tilde{\kappa}} = \left(\frac{\omega_i^*}{\tilde{\omega}}\right)^{\frac{1}{1-\mu_i}} \tag{42}$$

Similarly, the implicit solution of the factor price ratio (30) becomes:

$$\omega_i^* = c\left[\frac{\bar{L}}{\bar{K}} - \frac{1+c}{\tilde{\kappa}}\left(\frac{\omega_i^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_i}}\right]^{-1} \tag{43}$$

Further, the function $F(\cdot)$ in (31) becomes:

$$F_i\left(\omega_i^*; \mu_i, \frac{\bar{L}}{\bar{K}}, c, \tilde{\kappa}\right) = (\omega_i^*)^{-1} c + \frac{1+c}{\tilde{\kappa}}\left(\frac{\omega_i^*}{\tilde{\omega}}\right)^{-\frac{1}{1-\mu_i}} - \frac{\bar{L}}{\bar{K}} = 0 \tag{44}$$

It is immediate from (43) that the equilibrium factor price ratio is independent of μ iff $\tilde{\omega} = \omega_i^*$. From (42) we have that $\omega_i = \tilde{\omega}$ implies $L_i^{m*}/\bar{K} = \tilde{L}_i^m/\tilde{K}$ whereby $\nu = 1$ whenever optimal factor allocation to routine input production mimicks the allocation at the point of normalization.

3.1.6 A particular normalization: $\tilde{\kappa} = 1$

There exists one particular normalization of the CES production function for which $\varpi_i = \nu = 1$ at the point of normalization. From (40), the effective factor price ratio is $\varpi_i(\mu) = \tilde{\kappa}^{1-\mu}[\omega_i^*/\tilde{\omega}]$. Choosing $\tilde{\kappa} = 1$ at the point of normalization entails $\alpha_i = \alpha = (1+\tilde{\omega})^{-1}$, $A_i = A$, and $\varpi_i = 1$ whenever $\nu = 1$.[16] We plug these values into (43) to pin down the set of choices for initial endowments that are consistent with this normalization: $\tilde{L}/\tilde{K} > (1+c)$. We obtain the wage that for a given choice of endowments at the point of normalization equalizes the relative cost of labor in the two countries: $\tilde{\omega}_i(\tilde{L},\tilde{K};c) = c\left[(\tilde{L}/\tilde{K}) - (1+c)\right]^{-1} = \tilde{\omega}$. It is immediate that the relative price of the routine input is equalized in the two countries at the point of normalization (plug the values of α and A together with $\omega_i^* = \tilde{\omega}$ into (35)).

We now investigate how the relative wage changes when factor endowments deviate from the point of normalization. It is immediate from (43) that a shock to endowments that leaves relative endowments unchanged ($\bar{K}/\bar{L} = \tilde{K}/\tilde{L}$) leaves the relative wage unchanged and independent of μ. Thus, a proportional shock to factor endowments situates optimal routine input production on the ray from the origin to the point of normalization in the K-L_i^m plane, with factor allocation and factor prices independent of μ.

[16] For simplicity, we can always normalize $A = 1$ by defining $\tilde{M} = \tilde{L}^m = \tilde{K}$.

Consequently, we focus on endowment shocks that modify the capital-labor ratio in the economy relatively to the point of normalization. Without loss of generality, we fix the labor endowment $\bar{L} = \tilde{L}$ and consider shocks to the stock of capital: $\bar{K} \neq \tilde{K}$. As previously, we apply the implicit function theorem to $F(\cdot)$ in (**??**) to get:

$$\frac{\partial \omega_i^*}{\partial K} = -\frac{\partial F_i(\cdot)/\partial K}{\partial F_i(\cdot)/\partial \omega_i^*} > 0 \tag{45}$$

An increase (decrease) in the capital stock unambiguously increases (decreases) the relative wage ω_i^*. Consequently, the relative wage exceeds the relative wage at the point of normalization whenever the stock of capital exceeds the stock of capital at the point of normalization:

$$\begin{cases} \frac{\omega_i^*}{\tilde{\omega}} > 1, & \bar{K} > \tilde{K} \\ \frac{\omega_i^*}{\tilde{\omega}} = 1, & \bar{K} = \tilde{K} \\ \frac{\omega_i^*}{\tilde{\omega}} < 1, & \bar{K} < \tilde{K} \end{cases}$$

We have previously established that the relative wage is decreasing in μ whenever the effective cost of labor ϖ_i^* exceeds 1. With this normalization, $\varpi_i = \omega_i^*/\tilde{\omega}$. Hence, we reformulate our results in terms of shocks to endowments relatively to the point of normalization:

$$\begin{cases} \frac{\partial \omega_i^*}{\partial \mu} < 0, & \bar{K} > \tilde{K} \Leftrightarrow \frac{dv}{d\mu} > 0 \\ \frac{\partial \omega_i^*}{\partial \mu} = 0, & \bar{K} = \tilde{K} \Leftrightarrow \frac{dv}{d\mu} = 0 \\ \frac{\partial \omega_i^*}{\partial \mu} > 0, & \bar{K} < \tilde{K} \Leftrightarrow \frac{dv}{d\mu} < 0 \end{cases}$$

To sum up, a higher μ dampens the effect of any shock to factor endowments on the equilibrium relative wage.[17] Thus, if the shock to the stock of capital is positive, labor becomes more expensive than at the point of normalization in both countries, but less so in the high-μ country: $\tilde{\omega} < \omega_1^* < \omega_2^*$. If the shock to the stock of capital is negative, labor becomes less expensive than at the point of normalization in both countries, but less so in the high-μ country: $\tilde{\omega} > \omega_1^* > \omega_2^*$.

This dampening effect leads the high-μ country to specialize in non-routine production when labor becomes relatively scarce (through capital deepening) and to specialize in routine production when labor becomes relatively abundant. Indeed, the relative price of the routine input is increasing (decreasing) in μ whenever the stock of capital increases (decreases) relatively to the point of normalization:

$$\begin{cases} \frac{d(P_i^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} < 0 & \bar{K} < \tilde{K} \\ \frac{d(P_i^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} = 0 & \bar{K} = \tilde{K} \\ \frac{d(P_i^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} > 0 & \bar{K} > \tilde{K} \end{cases}$$

[17] Any given change in capital intensity leads to a smaller change in the marginal product of labor in the high-μ country because μ is inversely related to the cross-partial derivative of output with respect to K and L.

This result suffices to establish that the high-μ country is relatively non-routine labor abundant (33) under capital deepening: $(L_{a1}/S_1)^* > (L_{a2}/S_2)^*$.[18] The intuition behind this re-sult is the following. Consider a shock to technology in the low-μ country such that μ in-

creases. When μ goes up, the same quantity of inputs delivers more output in the routine sector ($M_1(w_2/r_2) > M_2(w_2/r_2)$). But labor is expensive relatively to the cost-minimizing factor combination in routine production because of the increase in μ. Consequently, labor is released from routine tasks, and this labor can only be absorbed in non-routine tasks whereby $M_1(\omega_2^*) \gg M_1(\omega_1^*)$. The price of labor goes down up to the point where extra labor absorbed in production of final goods is just enough to absorb excess labor released from routine input production. It follows that $L_1^{a*} \gg L_2^{a*}$. The ambiguity comes from the fact that the release of labor from routine input production does not suffice to prove that $M_1^* \leq M_2^*$. The high-μ country becomes non-routine abundant because the direct effect on labor allocation outweighs the indirect effect on routine input production: $L_1^{a*}/M_1^* \gg L_2^{a*}/M_2^*$.

3.1.7 Generalization: $\kappa \neq 1$

More generally, we can choose any $\kappa \neq 1$ at the point of normalization whereby $\varpi_i = 1$ when $\omega_i^* = \tilde{\omega}\tilde{\kappa}^{\mu_i - 1}$ and $\varpi_i \neq 1$ at the point of normalization defined by $v = 1 \Leftrightarrow \omega_i^* = \tilde{\omega}$. The distribution and productivity terms are now country-specific. We plug these values into (43) to pin down the set of feasible choices for initial endowments: $\tilde{L}/\tilde{K} > (1+c)/\tilde{\kappa}$. We obtain the wage that equalizes the relative wage in the two countries for a given choice of endowments at the point of normalization: $\tilde{\omega}(\tilde{L},\tilde{K};c) = c\left[(\tilde{L}/\tilde{K}) - (1+c)/\tilde{\kappa}\right]^{-1}$.[19] As previously, the relative price of the routine input is equalized in the two countries at the point of normalization (plug the values of α_i and A_i together with $\omega_i^* = \tilde{\omega}$ into (35)).

Again, we investigate how the relative wage changes when we deviate from the point of normalization. The derivative of the equilibrium wage with respect to the capital stock is positive whereby the following relationships continue to hold:

$$\begin{cases} \frac{\omega_i^*}{\tilde{\omega}} > 1, & \bar{K} > \tilde{K} \\ \frac{\omega_i^*}{\tilde{\omega}} = 1, & \bar{K} = \tilde{K} \\ \frac{\omega_i^*}{\tilde{\omega}} < 1, & \bar{K} < \tilde{K} \end{cases}$$

Further, the sign of $\partial(w_i/r_i)^*/\partial\mu$ is still determined by the sign of $\partial F_i(\cdot)/\partial\mu$ because

[18] This statement is equivalent to saying that the non-routine intensive good is relatively cheap in the high-μ

country: $P_{11}/P_{12} < P_{21}/P_{22}$.

[19] Equivalently, $\tilde{\omega} = c\tilde{K}\left[\tilde{L} - (1+c)\tilde{L}^m\right]^{-1}$.

$\partial F(\cdot)/\partial \omega_i^* < 0$. The latter is now directly determined by the wage relatively to the wage at the point of normalization:

$$\frac{\partial F_i(\cdot)}{\partial \mu} = -\ln\left(\frac{\omega_i^*}{\tilde{\omega}}\right)\frac{(1+c)}{\tilde{\kappa}(1-\mu)^2}\left(\frac{\omega_i^*}{\tilde{\omega}}\right)^{-\frac{1}{1-\mu}} \tag{46}$$

Thus, labor is relatively cheap in the high-μ country when the cost of labor increases relatively to the point of normalization. Further, labor is relatively expensive in the high-μ country when the cost of labor decreases relatively to the point of normalization:

$$\begin{cases} \frac{\partial \omega_i^*}{\partial \mu} < 0, & \left(\frac{\omega_i^*}{\tilde{\omega}}\right) > 1 \Leftrightarrow \bar{K} > \tilde{K} \Leftrightarrow \frac{d\nu}{d\mu} > 0 \\ \frac{\partial \omega_i^*}{\partial \mu} = 0, & \left(\frac{\omega_i^*}{\tilde{\omega}}\right) = 1 \Leftrightarrow \bar{K} = \tilde{K} \Leftrightarrow \frac{d\nu}{d\mu} = 0 \\ \frac{\partial \omega_i^*}{\partial \mu} > 0, & \left(\frac{\omega_i^*}{\tilde{\omega}}\right) < 1 \Leftrightarrow \bar{K} < \tilde{K} \Leftrightarrow \frac{d\nu}{d\mu} < 0 \end{cases}$$

A higher μ dampens the effect of any shock to factor endowments on the equilibrium relative wage.[20] Thus, if the shock to the stock of capital is positive, labor becomes more expensive than at the point of normalization in both countries, but less so in the high-μ country: $\tilde{\omega} < \omega_1^* < \omega_2^*$. If the shock to the stock of capital is negative, labor becomes less expensive than at the point of normalization in both countries, but less so in the high-μ country: $\tilde{\omega} > \omega_1^* > \omega_2^*$.

This dampening effect leads the high-μ country to specialize in non-routine production when labor becomes relatively scarce (through capital deepening) and to specialize in routine production when labor becomes relatively abundant. Indeed, the relative price of the routine input is increasing (decreasing) in μ whenever the stock of capital increases (decreases) relatively to the point of normalization:

$$\begin{cases} \frac{d(P^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} < 0 & \bar{K} < \tilde{K} \\ \frac{d(P^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} = 0 & \bar{K} = \tilde{K} \\ \frac{d(P^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} > 0 & \bar{K} > \tilde{K} \end{cases}$$

To sum up, the choice of $\tilde{\kappa}$ has no incidence on the mechanism at work. To simplify notation, we will henceforth work with the specific case of $\tilde{\kappa} = 1$.

3.2 Opening up to trade

3.2.1 The intuition

Opening up to trade amplifies differences in labor allocation to routine and non-routine tasks that were observed in autarky. The intuition is the following. Differences in capital-labor substitutability in the two countries lead to a wedge in the MPL/MPK ratio in the autarky

[20] Any given change in capital intensity leads to a smaller change in the marginal product of labor in the high-μ country because μ is inversely related to the cross-partial derivative of output with respect to K and L.

equilibrium which leads to a wedge in the relative autarky price of the two final goods. When the relative wage increases, the cost of labor allocated to non-routine tasks increases by more than the cost of the routine input because the latter uses both capital and labor. Consequently, the final good that requires more labor in non-routine tasks is relatively cheap in the country with the relatively low MPL/MPK ratio.

Trade equalizes the relative price of the two final goods by increasing the relative price of the good that was relatively cheap in autarky. The MPL/MPK ratio decreases in the country where it was relatively high, and it increases in the country where it was relatively low. The capital endowment is fixed by assumption. It follows that the only way to reduce (increase) the MPL/MPK ratio is to move labor into (out of) routine input production. Thus, the country that had a relatively high MPL/MPK ratio and, consequently, a relatively low price of the routine-intensive good, allocates more labor to routine input production. At the same time, the country that had a relatively low MPL/MPK ratio allocates more labor to non-routine tasks.

3.2.2 The illustration

The price of the final good is:

$$P_{ig} = \frac{w_i^{1-\beta_g} P_i^{m\beta_g}}{z_g \beta_g^{\beta_g}(1-\beta_g)^{1-\beta_g}}$$

We replace P_i^m by its value and rearrange the expression to get:

$$P_{ig} = \frac{w_i \left(\frac{r_i}{w_i}\right)^{\beta_g} \alpha_i^{-\frac{\beta_g}{\mu_i}} \left(1 + \left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right)^{-\frac{\beta_g(1-\mu_i)}{\mu_i}}}{A_i^{\beta_g} z_g \beta_g^{\beta_g}(1-\beta_g)^{1-\beta_g}}$$

The relative price of the two final goods is:

$$\frac{P_{i1}}{P_{i2}} = \frac{z_2 \beta_2^{\beta_2}(1-\beta_2)^{1-\beta_2}}{z_1 \beta_1^{\beta_1}(1-\beta_1)^{1-\beta_1} \alpha_i^{\frac{\beta_1-\beta_2}{\mu_i}} \left(\frac{w_i}{r_i}\right)^{\beta_1-\beta_2} A_i^{\beta_1-\beta_2} \left(1 + \left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right)^{\frac{(\beta_1-\beta_2)(1-\mu_i)}{\mu_i}}}$$

To simplify the expression, we use the normalization $\tilde{\kappa} = 1$ whereby $A_i = 1$ and $\alpha_i = (1+\tilde{\omega})^{-1}$ and further group all the country-invariant terms under the constant B. We have:

$$\frac{P_{i1}}{P_{i2}} = B(1+\tilde{\omega})^{\frac{\beta_1-\beta_2}{\mu_i}} \left[\omega_i \left(1 + \omega_i \left(\frac{\omega_i}{\tilde{\omega}}\right)^{-\frac{1}{1-\mu_i}}\right)^{\frac{(1-\mu_i)}{\mu_i}}\right]^{\beta_2-\beta_1}$$

Introducing ω_i into square brackets we get:

$$\frac{P_{i1}}{P_{i2}} = B(1+\tilde{\omega})^{\frac{\beta_1-\beta_2}{\mu_i}} \left[\omega_i^{\frac{\mu_i}{1-\mu_i}} + \tilde{\omega}^{\frac{1}{1-\mu_i}} \right]^{\frac{(\beta_2-\beta_1)(1-\mu_i)}{\mu_i}}$$

The derivative of the relative price wrt the relative wage ω_i is positive if good 1 is non-routine abundant ($\beta_1 < \beta_2$). Next, consider the relative price of the two final goods for the two countries:

$$\frac{P_{11}/P_{12}}{P_{21}/P_{22}} = (1+\tilde{\omega})^{\frac{(\mu_1-\mu_2)(\beta_2-\beta_1)}{\mu_1\mu_2}} \left[\omega_1^{\frac{\mu_1}{1-\mu_1}} + \tilde{\omega}^{\frac{1}{1-\mu_1}} \right]^{\frac{(\beta_2-\beta_1)(1-\mu_1)}{\mu_1}} \left[\omega_2^{\frac{\mu_2}{1-\mu_2}} + \tilde{\omega}^{\frac{1}{1-\mu_2}} \right]^{\frac{(\beta_1-\beta_2)(1-\mu_2)}{\mu_2}}$$

We use $\omega_2/\omega_1 = v$ to write:

$$\frac{P_{11}/P_{12}}{P_{21}/P_{22}} = (1+\tilde{\omega})^{\frac{(\mu_1-\mu_2)(\beta_2-\beta_1)}{\mu_1\mu_2}} \left[(\omega_2/v)^{\frac{\mu_1}{1-\mu_1}} + \tilde{\omega}^{\frac{1}{1-\mu_1}} \right]^{\frac{(\beta_2-\beta_1)(1-\mu_1)}{\mu_1}} \left[\omega_2^{\frac{\mu_2}{1-\mu_2}} + \tilde{\omega}^{\frac{1}{1-\mu_2}} \right]^{\frac{(\beta_1-\beta_2)(1-\mu_2)}{\mu_2}}$$

The above expression illustrates that any change in the relative price ratio can be studied as a function of the wedge in the relative wage of country 2 and country 1. It is immediate that the relative price of the non-routine intensive good is decreasing in v.

Suppose $v > 1$ in autarky. To equate the relative price of the non-routine intensive good in both countries, v must be reduced whereby ω_1 must go up. The latter can only occur if we move labor out of routine input production in country 1. Hence, country 1 specializes in the non-routine intensive good when the relative autarky price of this good is lower in country 1. Suppose $v < 1$ in autarky. To equate the relative price of the non-routine intensive good in both countries, v must increase whereby ω_2 must go up. The latter can only occur if we move labor out of routine input production in country 2. Hence, country 2 specializes in the non-routine intensive good when the relative autarky price of this good is lower in country 2.

3.2.3 Free Trade Equilibrium

The free trade equilibrium is a vector of allocations for consumers ($\hat{Q}_{ig}, i, g = 1, 2$), allocations for the firm ($\hat{K}_{ig}$, $\hat{L}^m_{ig}$, $\hat{L}^a_{ig}$, $\hat{M}_{ig}, i, g = 1, 2$), and prices ($\hat{w}_i$, $\hat{r}_i$, $\hat{P}^m_i$, $\hat{P}_g, i, g = 1, 2$) such that given prices consumer's allocation maximizes utility, and firms' allocations solve the cost minimization problem in each country, goods and factor markets clear: $\sum_i \hat{Q}_{ig} = \sum_i \hat{Y}_{ig}, g = 1, 2$; $\sum_g \hat{K}_{ig} = \bar{K}, i = 1, 2$; $\sum_g \hat{L}^a_{ig} + \hat{L}^m_{ig} = \bar{L}, i = 1, 2$; $\sum_g \hat{M}_{ig} = \hat{M}_i, i = 1, 2$.

Whenever both final goods are produced in both countries, firms' allocations satisfy: $\beta_g P_g z_g M_{ig}^{\beta_g-1} L^{a\,1-\beta_g}_{ig} = P^m_i$ and $(1-\beta_g) P_g z_g M_{ig}^{\beta_g} L^{a\,-\beta_g}_{ig} = w_i$. Further, from the ZPC, the price of each final good in each country is $P_{ig} = P^{m\beta_g}_i w_i^{1-\beta_g}/Z$ where $Z = z_g \beta_g^{\beta_g}(1-\beta_g)^{1-\beta_g}$. Prices are equalized through trade whereby: $(P^m_1/P^m_2)^{\beta_g} = (w_2/w_1)^{1-\beta_g}$. We solve for P^m_1/P^m_2 in one sector and plug the solution in the expression for the other sector to get:

$$\left(\frac{w_2}{w_1}\right)^{\frac{1-\beta_2}{\beta_2}} = \left(\frac{w_2}{w_1}\right)^{\frac{1-\beta_1}{\beta_1}}, \beta_2 \neq \beta_1 \Leftrightarrow w_2 = w_1 \tag{47}$$

As in the canonical HO model, trade leads to factor price equalization: the cost of labor and the cost of the routine input are equalized through trade. The feature specific to our model is that in general opening up to trade does not result in capital cost equalization. To see why, recall that firms' cost minimization in routine input production delivers (20). Given FPE, we have: $P^m = A_i^{-1}\left[\alpha_i^{\sigma_i} r_i^{1-\sigma_i} + (1-\alpha_i)^{\sigma_i} w^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}$. We use the normalization $\tilde{\kappa} = 1$ whereby $A_i = 1$ and $\alpha_i = (1+\tilde{\omega})^{-1}$ to simplify this expression and to solve for r_i in each country:

$$\begin{cases} r_1 = \left[(1+\tilde{\omega})^{\sigma_1} P^{m\,1-\sigma_1} - \tilde{\omega}^{\sigma_1} w^{1-\sigma_1}\right]^{\frac{1}{1-\sigma_1}} \\ r_2 = \left[(1+\tilde{\omega})^{\sigma_2} P^{m\,1-\sigma_2} - \tilde{\omega}^{\sigma_2} w^{1-\sigma_2}\right]^{\frac{1}{1-\sigma_2}} \end{cases}$$

The two expressions only differ by μ whereby in general $r_1 \neq r_2$.[21] Below we show that $r_1 = r_2$ iff $w/r_1 = w/r_2 = \tilde{\omega}$.

We connect the equilibrium relative price of the routine input and of labor to the allocation of resources to routine and non-routine tasks. Firm cost minimization in final goods' production delivers $\beta_g P_g z_g M_{ig}^{\beta_g} (L^a_{ig})^{1-\beta_g} = P^m M_{ig}$ and $(1-\beta_g) P_g z_g M_{ig}^{\beta_g} (L^a_{ig})^{1-\beta_g} = w L^a_{ig}$. Rearranging these two expressions and summing across countries delivers:

$$\begin{cases} P_g Y_{ig} = P^m M_{ig}/\beta_g \;\Leftrightarrow\; \sum_i Y_{ig} = \frac{P^m}{P_g \beta_g} \sum_i M_{ig} \\ P_g Y_{ig} = w L^a_{ig}/(1-\beta_g) \;\Leftrightarrow\; \sum_i Y_{ig} = \frac{w}{P_g(1-\beta_g)} \sum_i L^a_{ig} \end{cases}$$

First order conditions of the consumer problem in each country give:

$$\begin{cases} \theta_1 = \lambda_i P_1 Q_{i1} \\ \theta_2 = \lambda_i P_2 Q_{i2} \end{cases}$$

Summing the FOCs for each good in the two countries gives: $\theta_g/\lambda_1 + \theta_g/\lambda_2 = P_g(\sum_i Q_{ig})$. From goods' market clearing $\sum_i Q_{ig} = \sum_i Y_{ig}$. Plugging in the two expressions of $\sum_i Y_{ig}$ we get:

$$\begin{cases} \sum_i \frac{\theta_g}{\lambda_i} = \frac{P^m}{\beta_g} \sum_i M_{ig} \Leftrightarrow \sum_i \frac{1}{\lambda_i} \sum_g \beta_g \theta_g = P^m \sum_g \sum_i M_{ig} \Leftrightarrow \sum_i \frac{1}{\lambda_i} = P^m \frac{M_1^* + M_2^*}{\sum_g \beta_g \theta_g} \\ \sum_i \frac{\theta_g}{\lambda_i} = \frac{w}{(1-\beta_g)} \sum_i L^a_{ig} \Leftrightarrow \sum_i \frac{1}{\lambda_i} \sum_g (1-\beta_g)\theta_g = w \sum_g \sum_i L^a_{ig} \Leftrightarrow \sum_i \frac{1}{\lambda_i} = w \frac{L_1^* + L_2^*}{\sum_g (1-\beta_g)\theta_g} \end{cases}$$

[21] Expressions are more cumbersome if the general normalization $\kappa \neq 1$ is used, but the conclusion is unchanged.

Combining the above expressions delivers:

$$\frac{L_1^{a*}+L_2^{a*}}{M_1^*+M_2^*} = c\frac{P^m}{w} \tag{48}$$

Notice that the expression on the RHS can be written in two ways, depending on whether we use the expression of the price index in country 1 or in country 2. Replacing P^m by its value in each of the two countries gives:

$$A_1^{-1}\left[\alpha_1^{\frac{1}{1-\mu_1}}\left(\frac{w}{r_1}\right)^{\frac{\mu_1}{1-\mu_1}}+(1-\alpha_1)^{\frac{1}{1-\mu_1}}\right]^{-\frac{1-\mu_1}{\mu_1}} = A_2^{-1}\left[\alpha_2^{\frac{1}{1-\mu_2}}\left(\frac{w}{r_2}\right)^{\frac{\mu_2}{1-\mu_2}}+(1-\alpha_2)^{\frac{1}{1-\mu_2}}\right]^{-\frac{1-\mu_2}{\mu_2}}$$

We use the normalization $\tilde{\kappa}=1$ whereby $A_i=1$ and $\alpha_i=(1+\tilde{\omega})^{-1}$ to get:

$$(1+\tilde{\omega})^{\frac{1}{\mu_1}}\left[\left(\frac{w}{r_1}\right)^{\frac{\mu_1}{1-\mu_1}}+\tilde{\omega}^{\frac{1}{1-\mu_1}}\right]^{-\frac{1-\mu_1}{\mu_1}} = (1+\tilde{\omega})^{\frac{1}{\mu_2}}\left[\left(\frac{w}{r_2}\right)^{\frac{\mu_2}{1-\mu_2}}+\tilde{\omega}^{\frac{1}{1-\mu_2}}\right]^{-\frac{1-\mu_2}{\mu_2}} \tag{49}$$

It is easy to check that setting $w/r_1 = w/r_2 = \tilde{\omega}$ solves (49). As expected, at the point of normalization, resource allocation and equilibrium relative factor prices are the same in both countries. In all other cases we can solve for the equilibrium factor price ratio in one country as a function of the factor price ratio in the other country:

$$\frac{w}{r_1} = \left[(1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}}\left(\left(\frac{w}{r_2}\right)^{\frac{\mu_2}{1-\mu_2}}+\tilde{\omega}^{\frac{1}{1-\mu_2}}\right)^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}}-\tilde{\omega}^{\frac{1}{1-\mu_1}}\right]^{\frac{1-\mu_1}{\mu_1}}$$

$$\omega_1^* = \left[(1+\tilde{\omega})^{\frac{\sigma_2-\sigma_1}{\sigma_2-1}}\left(\left(\frac{w}{r_2}\right)^{\sigma_2-1}+\tilde{\omega}^{\sigma_2}\right)^{\frac{\sigma_1-1}{\sigma_2-1}}-\tilde{\omega}^{\sigma_1}\right]^{\frac{1}{\sigma_1-1}} \tag{50}$$

$$\frac{w}{r_2} = \left[(1+\tilde{\omega})^{\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}}\left(\left(\frac{w}{r_1}\right)^{\frac{\mu_1}{1-\mu_1}}+\tilde{\omega}^{\frac{1}{1-\mu_1}}\right)^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}}-\tilde{\omega}^{\frac{1}{1-\mu_2}}\right]^{\frac{1-\mu_2}{\mu_2}}$$

$$\omega_2^* = \left[(1+\tilde{\omega})^{\frac{\sigma_1-\sigma_2}{\sigma_1-1}}\left(\left(\frac{w}{r_1}\right)^{\sigma_1-1}+\tilde{\omega}^{\sigma_1}\right)^{\frac{\sigma_2-1}{\sigma_1-1}}-\tilde{\omega}^{\sigma_2}\right]^{\frac{1}{\sigma_2-1}} \tag{51}$$

Next, we work with the LHS of (48). We use firm cost minimization in routine input production together with factor market clearing to rewrite the LHS as a function of the equilibrium factor price ratio and factor endowments. Capital market clearing (see 73) delivers:

$$M_i^* = A_i\alpha_i^{1/\mu_i}\bar{K}\left[1+(\omega_i^*)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{1/\mu_i}$$

(52)

Labor market clearing delivers $L^{a*}_i = \bar{L} - L^{m*}_i$ while cost minimization in routine input production and the total capital stock determine labor allocation to routine tasks:

$$L^{m*}_i(M^*_i) = (\omega^*_i)^{\frac{-1}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} \tilde{\nu} \tag{53}$$

We simplify (52) and (53) with the normalization $\tilde{\kappa} = 1$ and rearrange to get:

$$\frac{L^{a*}_1 + L^{a*}_2}{M^*_1 + M^*_2} = \frac{2\frac{\bar{L}}{\tilde{K}} - \left(\frac{\omega^*_1}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_1}} - \left(\frac{\omega^*_2}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_2}}}{(1+\tilde{\omega})^{\frac{-1}{\mu_1}}\left(1+\omega^*_1\left(\frac{\omega^*_1}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_1}}\right)^{1/\mu_1} + (1+\tilde{\omega})^{\frac{-1}{\mu_2}}\left(1+\omega^*_2\left(\frac{\omega^*_2}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_2}}\right)^{1/\mu_2}} \tag{54}$$

We solve for the price ratio in each country by plugging the expressions for the LHS and the RHS into (48) and plugging the expression of the factor price ratio as a function of the factor price ratio in the other country. To simplify notation, we define $\Omega_i = (\omega^*_i)^{\frac{\mu_i}{1-\mu_i}} + \tilde{\omega}^{\frac{1}{1-\mu_i}}$.

For the high-μ country we get:

$$\frac{2\frac{\bar{L}}{\tilde{K}} - \left(\frac{\omega^*_1}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_1}} - \tilde{\omega}^{\frac{1}{1-\mu_2}}\left((1+\tilde{\omega})^{\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}}\Omega_1^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}} - \tilde{\omega}^{\frac{1}{1-\mu_2}}\right)^{\frac{-1}{\mu_2}}}{(1+\tilde{\omega})^{\frac{-1}{\mu_1}}\left(1+\omega^*_1\left(\frac{\omega^*_1}{\tilde{\omega}}\right)^{\frac{-\mu_1}{1-\mu_1}}\right)^{\frac{1}{\mu_1}} + (1+\tilde{\omega})^{\frac{-(1-\mu_1)}{\mu_1(1-\mu_2)}}\Omega_1^{\frac{(1-\mu_1)}{\mu_1(1-\mu_2)}}\left((1+\tilde{\omega})^{\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}}\Omega_1^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}} - \tilde{\omega}^{\frac{1}{1-\mu_2}}\right)^{\frac{1}{\mu_2}}}$$
$$= c(1+\tilde{\omega})^{\frac{1}{\mu_1}}\Omega_1^{\frac{-(1-\mu_1)}{\mu_1}}$$

For the low-μ country we get:

$$\frac{2\frac{\bar{L}}{\tilde{K}} - \left(\frac{\omega^*_2}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_2}} - \tilde{\omega}^{\frac{1}{1-\mu_1}}\left((1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}}\Omega_2^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}} - \tilde{\omega}^{\frac{1}{1-\mu_1}}\right)^{\frac{-1}{\mu_1}}}{(1+\tilde{\omega})^{\frac{-1}{\mu_2}}\left(1+\omega^*_2\left(\frac{\omega^*_2}{\tilde{\omega}}\right)^{\frac{-\mu_2}{1-\mu_2}}\right)^{\frac{1}{\mu_2}} + (1+\tilde{\omega})^{\frac{-(1-\mu_2)}{\mu_2(1-\mu_1)}}\Omega_2^{\frac{(1-\mu_2)}{\mu_2(1-\mu_1)}}\left((1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}}\Omega_2^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}} - \tilde{\omega}^{\frac{1}{1-\mu_1}}\right)^{\frac{1}{\mu_1}}}$$
$$= c(1+\tilde{\omega})^{\frac{1}{\mu_2}}\Omega_2^{\frac{-(1-\mu_2)}{\mu_2}}$$

Rearranging and simplifying this expression, the implicit solution for the high-μ country is:

$c(1+\tilde{\omega})^{-1}$

$$F_1\left(\omega_1;\mu_1,\mu_2,\frac{\bar{L}}{\bar{K}},c\right)=c\,(\omega^*)^{-\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}}\frac{\bar{L}}{\bar{K}}+(c+1)\left(\frac{\omega_1^*}{\tilde{\omega}}\right)^{\frac{1}{1-\mu_1}}-2\frac{\bar{L}}{\bar{K}}+\frac{\Omega_1^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}}+\tilde{\omega}^{\frac{1}{1-\mu_2}}}{(1+\tilde{\omega})^{\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}}\Omega_1^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}}-\tilde{\omega}^{\frac{1}{1-\mu_2}}}=0$$

Rearranging and simplifying this expression, the implicit solution for the low-μ country is:

$$F_2(\omega_2;\mu_1,\mu_2,\frac{\bar{L}}{\bar{K}},c) = c\,(\omega^*)^{-\frac{1}{1-\mu_2}} + (c+1)\left(\frac{\omega_2^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_2}} - 2\frac{\bar{L}}{\bar{K}} + \frac{\left[c(1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}}\Omega_2^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}} + \tilde{\omega}\right]^{\frac{1}{1-\mu_1}}}{\left[(1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}}\Omega_2^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}} - \tilde{\omega}\right]^{\frac{1}{1-\mu_1}}}^{-\frac{1}{\mu_1}} = 0$$

The first two terms replicate the analogous expression for the autarky equilibrium (31) while the third term now takes into account factor endowments in both countries. The fourth term is specific to the FTE: it accounts for the difference in capital-labor substitutability.

We can rewrite these expressions as a function of σ. In the high-σ country we get:

$$F_1(\cdot) = c(\omega^*)^{-1} + (c+1)\left(\frac{\omega_1^*}{\tilde{\omega}}\right)^{-\sigma_1} - 2\frac{\bar{L}}{\bar{K}} + \frac{\left[c(1+\tilde{\omega})^{\frac{\sigma_1-\sigma_2}{\sigma_1-1}}(\omega_1^*)^{\sigma_1-1} + \tilde{\omega}\right]^{\frac{\sigma_1}{\sigma_2}\frac{\sigma_2-1}{\sigma_1-1}} + \tilde{\omega}}{\left[(1+\tilde{\omega})^{\frac{\sigma_1-\sigma_2}{\sigma_1-1}}(\omega_1^*)^{\sigma_1-1} + \tilde{\omega}\right]^{\frac{\sigma_2-1}{\sigma_1-1}} - \tilde{\omega}^{\sigma_2}}^{\frac{\sigma_2}{\sigma_2-1}} = 0$$

In the low-σ country we get:

$$F_2(\cdot) = c(\omega_2^*)^{-1} + (c+1)\left(\frac{\omega_2^*}{\tilde{\omega}}\right)^{\sigma_2} - 2\frac{\bar{L}}{\bar{K}} + \frac{\left[c(1+\tilde{\omega})^{\frac{\sigma_2-\sigma_1}{\sigma_2-1}}(\omega_2^*)^{\sigma_2-1} + \tilde{\omega}\right]^{\frac{\sigma_2}{\sigma_1}\frac{\sigma_1-1}{\sigma_2-1}} + \tilde{\omega}}{\left[(1+\tilde{\omega})^{\frac{\sigma_2-\sigma_1}{\sigma_2-1}}(\omega_2^*)^{\sigma_2-1} + \tilde{\omega}\right]^{\frac{\sigma_1-1}{\sigma_2-1}} - \tilde{\omega}^{\sigma_1}}^{\frac{\sigma_1}{\sigma_1-1}} = 0$$

4 Estimation

Our model predicts that low-σ countries export relatively more in routine intensive sectors. In this section we show that routine intensity is a strong predictor of the pattern of trade. We also connect the routine intensity of exports to country characteristics that may correlate with the institutional efficiency with which countries reallocate labor across tasks, i.e. with the magnitude of adjustment costs incurred by firms and workers in this process.

We document that countries with more flexible labor markets and higher workforce ability have a comparative advantage in sectors that use nonroutine labor more intensively. These findings motivate the microfoundations for country-level differences in the elasticity of substitution between capital and labor in routine tasks that we develop in the final section of the paper.

Our empirical strategy follows the two-step approach of Costinot (2009). In the first step, we retrieve the pattern of comparative advantage on the routine dimension and report the ranking of countries with respect to the routine intensity of their exports. In the second step, we regress the obtained ranking on country characteristics that may correlate with the efficiency of labor reallocation across tasks. We thus identify the institutional characteristics that contribute to determine the routine intensity of exports, namely the stringency of employment protection legislation (EPL) and the quality of the educational system.

4.1 Data sources

We work with bilateral sectoral trade data at the HS 2-digit level. Our sample covers the 19 biggest exporters i and the 34 biggest importers j in 2000-2006.

The key sectoral characteristic of interest for our exercise is the intensity with which the sector uses the routine tasks (parameter β_g of the model). We construct a proxy for the sectoral variation in β_g that we denote r_g by matching the ranking of routine intensity constructed by Autor et al. (2003) for 140 U.S. census industries to the HS 2-digit classification. The ranking of Autor et al. (2003) is based on the weighted average of the routine intensity of occupations in each industry, with the weights given by the employment shares of occupations in the industry in 1977. We expect this ranking to capture the technological features that determine the routine intensity of the industry because the data on employment used to construct the ranking pre-dates the recent process of automation through computerization.[22]

We consider several dimensions of country-level endowments I_i that may help to explain specialization in routine intensive goods (or, equivalently, lower capital-labor substitutability in routine production). We follow the labor literature in evaluating the role of labor market institutions, namely the stringency of EPL and the unionization rate.[23] We follow Costinot (2009) in evaluating the role of institutional quality with the 'rule of law' index.[24]

We also evaluate the role of behavioral norms of the workforce with help of the 'ability to perform' and the 'internal mobility' variables. As explained in Costinot (2009), the former is a synthetic index of worker attributes developed by the Business Environment Risk Intelligence (B.E.R.I) S.A. that combines work ethic, the quality of human capital, and physical characteristics such as healthiness. The latter is computed as the fraction of the population residing in a different region than their place of birth and is a rather coarse measure of workforce mobility.

[22] As shown by Autor et al. (2003), routine intensive industries replaced labor more intensively with machines and increased by more their demand for nonroutine labor. As the ranking of routine intensity is based on effective employment shares, it is sensitive to more intensive automation in routine intensive industries.

[23] Data on EPL stringency is taken from the OECD. The rate of unionization corresponds to the fraction of the workforce affiliated to a trade union.

[24] The most recent data: http://info.worldbank.org/governance/wgi/index. We use the ranking in the mid-2000s.

4.2 Results

In the first step we regress the log of bilateral sectoral exports X_{gijt} on pair fixed effects τ_{ij}, destination-sector fixed effects τ_{gj} and the interaction between the ranking of sectoral routineness r_g and exporter fixed effects γ_i.[25]

$$\ln X_{gijt} = \tau_{ij} + \tau_{gj} + \gamma_i r_g + \varepsilon_{gijt} \tag{55}$$

The comparative advantage in the routine dimension is captured by the ranking of $\hat{\gamma}_i$: high $\hat{\gamma}_i$'s pick up higher exports in high-r_g sectors. Table 5 reports our results for the sample of the 19 biggest world exporters in col. 3 and for 25 European countries in col. 5. We mark in *italic* the countries specialized in routine-intensive sectors - such as Italy or Portugal - and in ***italic*** those specialized in nonroutine intensive sectors - such as Germany or Sweden. Routineness is a strong predictor of specialization even for countries at a similar level of development.

In the second step we regress the estimated ranking of exports' routineness $\hat{\gamma}_i$ on each of the institutional dimensions I_i.

$$\ln \hat{\gamma}_i = \delta_0 + \delta_1 I_i + \varepsilon_i \tag{56}$$

The coefficient of interest is δ_1 which we expect to be negative for the normative and the institutional quality dimensions but positive for the stringency of EPL. The coefficient on the unionization rate is more difficult to sign. It is expected to be negative if higher unionization maps into low bargaining frictions but positive if higher unionization maps into high worker bargaining power.

Our results for the second step are reported in Table 6. The most important variable that singlehandedly explains 68% of the variation in the $\hat{\gamma}_i$ ranking is workforce ability. The second relevant variable is the rule of law that explains about 30% of the variation. We interpret these results as indicating that normative and behavioral dimensions likely contribute to determine both the magnitude of bargaining frictions and the level of general human capital that in turn determines the magnitude of adjustment costs on the side of the worker.

Labor market institutions do not help to explain the routine intensity of exports in our global sample. However, they do contribute to determining differences in the specialization of European countries. Our results suggest that higher costs of labor adjustment on the firm side, such as firing and hiring costs, lead to specialization of the country in more routine intensive sectors.

[25] The analogus equation in Costinot (2009) is (15).

Table 5: The estimated routine intensity of exports ($\hat{\gamma}_i$)

Rank	Main exporters	coef in 2000-2006	within Europe	coef in 2000-2006
1	*Thailand*	6.62***	*Lithuania*	6.12***
2	*Italy*	3.57**	*Latvia*	3.84**
3	*Canada*	3.40*	*Greece*	3.83**
4	China	2.59	*Portugal*	3.37*
5	*Spain*	2.45***	*Romania*	3.13*
6	Malaysia	1.26	*Cyprus*	2.79*
7	Austria	.42	*Bulgaria*	2.64**
8	France	.20	*Italy*	2.55**
9	Mexico	.17	Denmark	2.41
10	Netherlands	-.19	Malta	2.12
11	USA	-.63	*Spain*	1.41**
12	South Korea	-.71	Poland	.31
13	***UK***	-1.20***	Hungary	-.33
14	Belgium	-1.51	Slovakia	-.85
15	***Singapore***	-2.02***	France	-.91
16	***Sweden***	-2.92***	Slovenia	-.99
17	***Germany***	-3.07***	Austria	-1.21
18	***Switzerland***	-3.19***	Netherlands	-1.32
19	***Japan***	-5.45***	Czech Republic	-1.38
20			***Belgium-Luxembourg***	-1.71**
21			***Germany***	-1.73***
22			***UK***	-2.39***
23			Ireland	-2.77
24			***Sweden***	-4.18***
25			***Finland***	-7.09***

Further, the rate of unionization appears to capture lower bargaining frictions and, possibly, lower coordination costs associated to labor reallocation across tasks.

These results motivate our approach to microfounding our baseline model. In the next section we seek to pin down the institutional and normative sources of σ-variation across countries. We show that institutions and behavioral norms that help to reduce the cost of labor reallocation lead to higher capital intensity in routine tasks, a relatively low cost of nonroutine labor, and a comparative advantage in nonroutine intensive sectors.

Table 6: Institutional determinants of specialization in routine intensive sectors

Characteristic	Global unweighted	Global weighted	Within EU unweighted	Within EU weighted
Ability to perform	-0.827*** (6.06)	-0.775*** (5.06)	-0.666** (2.36)	-0.634** (2.17)
Internal migration	-0.300 (1.04)	-0.330 (1.16)	-0.086 (0.35)	-0.131 (0.53)
Rule of law	-0.543*** (2.67)	-0.563*** (2.81)	-0.585** (3.46)	-0.605** (3.64)
Unionization rate	-0.127 (0.46)	-0.131 (0.48)	-0.404* (1.82)	-0.402* (1.81)
Strictness of EPL	0.306 (1.29)	0.290 (1.21)	0.383* (1.81)	0.338* (1.56)
Max. observations	19	19	25	25

Note: we report standardized beta coefficients which measure effects in standard errors and *t*-statistics in brackets.
Note: weight is the inverse of the standard error of the first stage regression used to get the dependent variable.

5 Microfoundation of variation in K-L substitutability

In this section we show that institutional characteristics that predict countries' specialization according to the sectoral ranking of routine intensity can effectively lead to differences in perceived capital-labor substitutability in routine production that we posited in our baseline model. We start by reporting the main findings of the recent literature on the linkages between the institutional characteristics of the labor market and the adjustment of the economy to structural change. We then summarize our approach to microfounding differences in capital-labor substitutability across countries.

5.1 Labor market institutions and adjustment to structural change

An important stream of the recent labor literature documents that adjustment costs associated to the reallocation of workers across occupations are non-negligible for the median worker and strongly heterogeneous across workers. Dix-Carneiro (2014) finds that the median cost of switching jobs for Brazilian workers amounts to 1.4-2.7 times the average annual wage.[26] Dix-Carneiro (2014) shows that cost variability across workers is attributable to skills, age, initial specialization, and experience accumulated in the job. For the U.S. market, Autor et al. (2014) find that adjustment costs may be prohibitively high for the less skilled and the less young and lead to their permanent exit from the labor force.[27]

A key insight of this literature is that the cost of switching occupations is not fully determined by the cost of looking for a job or of moving to a new location. Rather, the bulk of the adjustment cost is attributable to the loss of firm- or occupation-specific human capital. Working with (respectively) Brazilian and U.S. data, Dix-Carneiro (2014) and Autor et al. (2014) explain the positive relationship between the magnitude of the adjustment cost and the distance from the initial to the final occupations by the loss of non-transferable human capital. For the Danish market, Ashournia (2015) documents that the loss of industry-specific human capital constitutes a substantial fraction of reallocation costs.

Although we acknowledge the importance of adjustment cost variability across workers, our focus is on institutional characteristics that determine the country-specific component of adjustment costs common to all workers. Specifically, we seek to quantify the contribution

[26] The seminal paper by Artuç et al. (2010) reports higher median costs but has a coarser approach to capturing differences in worker characteristics.

[27] Pierce and Schott (2016) report that 1/3 of workers who lost employment in U.S. manufacturing as a consequence of import competition from China transition to inactivity while 1/3 switches to services.

of institutional determinants of reallocation costs to the pattern of trade. We put forward two candidate institutional characteristics which may result in different average levels of transferable skills in the labor force and, subsequently, different per worker magnitudes of retraining costs: the quality of the educational system and the flexibility of labor market institutions (LMIs).

It is immediate that a less efficient educational system may result in a lower level of general human capital. As shown by Wasmer (2006), stringent labor market regulations may also result in a lower level of general human capital. Stringent LMIs are captured through high firing costs in Wasmer (2006). Their direct effect is to increase the cost of labor adjustment on the firm side and to reduce the separation rate in the economy. The increase in the expected duration of employment gives an incentive to workers to accumulate specific human capital endogenously increasing the cost of switching occupations on the worker side. This indirect effect leads to relatively high retraining costs and low job turnover.

Several papers put forward that stringent LMIs reduce the speed of adjustment of the economy to structural change. Wasmer (2006) demonstrates that economies with rigid LMIs perform relatively better in the steady state because workers are more productive in their jobs but have prolonged and costly transition periods. Kambourov (2009) shows that high firing costs slow down the process of worker reallocation to comparative advantage activities in an economy that opens up to trade and result in a sizeable reduction of the gains from trade.[28] Artuç et al. (2015) estimate the magnitude of switching costs for workers in a set of countries and document that countries with relatively high switching costs adjust more slowly to trade shocks.

Several other papers argue that LMIs co-determine the pattern of specialization. Tang (2012) derives the comparative advantage implications of reinforced worker incentives to accumulate firm-specific human capital.[29] Tang (2012)'s model predicts that stringent LMIs confer a comparative advantage in sectors that require intensive use of specific human capital. Tang (2012) measures the sectoral intensity of specific human capital use by estimating the sectoral return to tenure. Connecting the pattern of trade to the obtained sectoral ranking, Tang (2012) finds that countries with rigid LMIs export more in sectors with higher returns to tenure.

Even closer to our research focus are the papers by Cuñat and Melitz (2012) and Bartelsman et al. (2016) who look at the impact of stringent LMIs from the perspective of the firm. Cuñat and Melitz (2012) show that higher costs of labor adjustment confer a relative cost disadvantage in volatile sectors, with volatility defined in terms of the variance of firm-specific productivity

[28] Coşar (2013) finds that active (passive) labor market policies speed up (slow down) labor reallocation.
[29] Acharya et al. (2013) argues that higher EPL induces workers to engage in higher risk innovative projects.

shocks.[30] Bartelsman et al. (2016) connect the stringency of LMIs to reduced incentives to invest in risky technology by showing that EPL is akin to a distortive tax on risky investment.[31] The authors show that industries characterized by a greater degree of dispersion in labor productivity are also characterized by more intensive ICT usage and argue that the recent process of technological change through innovations in ICT corresponded to such high-risk high-return technology. Consistently with the predictions of the model, Bartelsman et al. (2016) document that countries with more stringent labor market regulations adopted ICT less intensively in the mid-1990s and specialized in less ICT-intensive industries.

5.2 σ is reconductible to the magnitude of labor adjustment costs

Overall, the literature summarized in the previous subsection suggests there is a linkage between labour market institutions, the set of skills that workers choose to acquire, the type of investment that firms choose to implement, and the equilibrium allocation of resources to different sectors of the economy. Our contribution to this line of work consists in explicitly connecting the level of labor adjustment costs to the magnitude of the parameter that captures capital-labor substitutability in the canonical CES production function.

Consider the definition of capital-labor substitutability: σ captures the percentage increase in the capital-labor ratio that follows a one percent increase in the relative cost of labor. We put forward that there may be a country-specific wedge between the underlying technological parameter common to all countries that captures how firms would adjust the capital-labor ratio in the absence of labor adjustment costs and the measured capital-labor substitutability that captures how firms effectively adjust the capital-labor ratio. A given shock to the relative price of labor translates into a smaller change of the capital-labor ratio when there is a cost for the firm of adjusting the labor input. Countries characterized by relatively high labor adjustment costs have a relatively low sensitivity to changes in the relative cost of labor.

The line of argument is as follows. We start from the production function in Autor et al. (2003) (section 2). Capital and routine labor are perfect substitutes while capital and abstract labor are imperfect substitutes. The justification for perfect substitutability in the routine tasks is that once the machine exists, both labor and the machine have the capability to accomplish the routine task (example: count coins), but their efficiency in the task may differ.

We consider some initial allocation of labor to routine and non-routine tasks for some initial

[30] Cuñat and Melitz (2012) proxy sectoral volatility with the standard deviation of firm-specific growth rates.

[31] EPL increases the cost of downsizing (exit) and reduces the expected return to investment in risky technology.

state of technology. We then compute the change in the labor input in the routine task that takes place following a positive shock to the efficiency of capital while keeping wages fixed.

The full effect of the technological shock would be to get rid of all labor in the routine task if labor productivity were the same for all units employed in the routine task and there were no adjustment costs incurred by the firm in laying off workers. In reality, the effect of this technological shock on the capital-labor ratio in routine production will be reduced because of labor adjustment costs incurred by the firm such as severance payments. The labor input is reduced by less because each unit of labor replaced with capital is associated to a severance payment, and these payments are likely to be a convex function of the number of laid-off workers. Severance payments increase the effective cost of the more efficient capital and reduce the sensitivity of the capital-labor ratio to changes in the relative factor price.

Measured capital-labor substitutability is decreasing in the degree of convexity of the severance payment function. An intuitive way of justifying the convexity of the severance payment function is to consider that workers are heterogeneous in the retraining costs required to reallocate them from the routine to the noroutine tasks. As more workers are laid off, the retraining cost per worker is increasing at an increasing speed. Thus, technological upgrading shifts labor out of a subset of routine tasks but labor eviction from such tasks is gradual because routine workers differ in the amount of training they require to perform the nonroutine task, and the institutional set-up that defines which agents bear this retraining cost determines the magnitude of labor market frictions that slow down the process of labor reallocation.

The final building block is to spell out how countries differ. One simple way of generating differences in the convexity of the retraining cost function is to consider intrinsic differences in the quality of schooling. Countries with higher level and lower variance of initially acquired human capital will have lower level and variance of re-training needs. Another way of generating differences in labor adjustment costs across countries is to consider that certain countries provide more generous financial support to employers who bear the re-training costs. In the latter case, even if the underlying convexity of the retraining cost function is common to the two countries, the effective convexity is lower in the country in which the government participates in retraining more intensively.

6 Conclusion

In this paper, we pin down a new mechanism behind comparative advantage by pointing out that countries may differ in their ability to adjust to technological change.

We take stock of the pattern extensively documented in the labor literature whereby more efficient machines displace workers from codifiable (routine) tasks. Our hypothesis is that labor reallocation across tasks is subject to frictions and that these frictions are country-specific. We incorporate task routineness into a canonical 2-by-2-by-2 Heckscher-Ohlin model. The key feature of our model is that factor endowments are determined by the equilibrium allocation of labor to routine and non routine tasks. Our model predicts that countries which facilitate labor reallocation across tasks become relatively abundant in non routine labor and specialize in goods that use non routine labor more intensively.

We document that the ranking of countries with respect to the routine intensity of their exports is strongly connected to two institutional aspects: labor market institutions and behavioral norms in the workplace. We proceed to develop microfoundations (in a non-formal way) which help to explain why the parameter that captures capital-labour substitutability and is generally perceived as an exogenous characteristic of the production technology may in fact be determined by the institutional environment.

Specifically, we show that any type of institutional characteristic which increases the cost of adjusting the labour input - such as the rigidity of labour market institutions or the lack of efficiency of the public administration in implementing active labour market policies - may increase the shadow cost of switching to more productive capital. Any given change in the relative cost of labour will result in a smaller change in the relative capital-labour ratio in routine production in a highly frictional environment and result in a lower perceived capital-labour substitutability in routine production.

Our results pin down a new linkage between institutions and the pattern of trade while showing that specific institutional characteristics facilitate the adjustment of the economy to the process of structural change. Our results have strong policy implications because they illustrate that governments have a key role to play in ensuring that the process of labour reallocation from tasks that are substitutable with machines to tasks that are complementary with machines proceeds quickly and smoothly. Indeed, workers are shown to benefit relatively more from the process of technological change and from trade integration in institutional environments that succeed in reducing the costs of labour reallocation across tasks.

References

Acemoglu, D. and D. H. Autor (2011). Skills, tasks and technologies: Implications for employment and earnings. In O. Ashenfelter and D. E. Card (Eds.), *Handbook of Labor Economics*. Amsterdam: Elsevier B.V.

Acemoglu, D. and P. Restrepo (2016). The race between machine and man: Implications of technology for growth, factor shares and employment. *NBER Working Paper Series* (22252).

Acharya, V., R. Baghai, and K. Subramanian (2013). Labor laws and innovation. *Journal of Law and Economics 56*, 997–1037.

Artuç, E., S. Chaudhuri, and J. McLaren (2010). Trade shocks and labor adjustment: A structural empirical approach. *American Economic Review 100*(3), 1008–1045.

Artuç, E., D. Lederman, and G. Porto (2015). A mapping of labor mobility costs in the developing world. *Journal of International Economics 95*, 28–41.

Ashournia, D. (2015). Labour market effects of international trade when mobility is costly. *Oxford University Department of Economics Discussion Paper* (751).

Autor, D. H. and D. Dorn (2013). The growth of low-skill service jobs and the polarization of the us labor market. *American Economic Review 103*(5), 1553–1597.

Autor, D. H., D. Dorn, G. H. Hanson, and J. Song (2014). Trade adjustment: Worker-level evidence. *The Quarterly Journal of Economics 129*(4), 1799–1860.

Autor, D. H., F. Levy, and R. J. Murnane (2003). The skill content of recent technological change: An empirical exploration. *Quarterly Journal of Economics 118*(4), 1279–1333.

Bartelsman, E., P. Gautier, and J. De Wind (2016). Employment protection, technology choice, and worker allocation. *International Economic Review 57*(3).

Bombardini, M., G. Gallipoli, and G. Pupato (2012). Skill dispersion and trade flows. *American Economic Review 102*(5), 2327–2348.

Chor, D. (2010). Unpacking sources of comparative advantage: A quantitative approach. *Journal of International Economics 82*, 152–167.

Coşar, K. (2013). Adjusting to trade liberalization: Reallocation and labor market policies. *mimeo*.

Costinot, A. (2009). On the origins of comparative advantage. *Journal of International Economics 77*, 255–264.

Cuñat, A. and M. Melitz (2012). Volatility, labor market flexibility, and the pattern of comparative advantage. *Journal of the European Economic Association 10*, 225–254.

Dix-Carneiro, R. (2014). Trade liberalization and labor market dynamics. *Econometrica 82*(3), 825–855.

Goos, M., A. Manning, and A. Salomons (2014). Explaining job polarization: Routine-biased technological change and offshoring. *American Economic Review 104*(8), 2509–2526.

Harrigan, J., A. Reshef, and F. Toubal (2016). The march of the techies: Technology, trade, and job polarization in france. *NBER Working Paper series* (22110).

Kambourov, G. (2009). Labor market regulations and the sectoral reallocation of workers: the case of trade reforms. *Review of Economic Studies 76*(4), 1321–1358.

Klump, R. and O. De La Grandville (2000). Economic growth and the elasticity of substitution: Two theorems and some suggestions. *The American Economic Review 90*(1), 282–291.

Klump, R., P. McAdam, and A. Willman (2012). The normalized ces production function: Theory and empirics. *Journal of Economic Surveys 26*(5), 769–799.

Nunn, N. and D. Trefler (2014). Domestic institutions as a source of comparative advantage. In G. Gopinath, E. Helpman, and K. Rogoff (Eds.), *Handbook of International Economics*, Volume 4. North Holland.

Pierce, J. R. and P. Schott (2016). The surprisingly swift decline in US. *American Economic Review 106*(7), 1632–1662.

Tang, H. (2012). Labor market institutions, firm-specific skills, and trade patterns. *Journal of International Economics 87*, 337–351.

Wasmer, E. (2006). General versus specific skills in labor markets with search frictions and firing costs. *The Quarterly Journal of Economics 96*(3), 811–831.

Appendices

A Characterization of the average cost function

A.1 AC minimization is the only solution to the macro problem

We can show that the choice of routine input production at which the average cost function is minimized provides the unique solution to the inner and outer problem defined in sec.3.1. To see this, consider any point on the marginal cost function (8) in choosing the quantity of the routine input M_i. The FOC for the inner problem is $MC_i^m = P_i^m$. Further, we know from the outer problem that (28) must hold. Hence, we combine (8) and (28), simplify the resulting expression for w_i and M_i, and use $c = \sum_n \theta_n(1-\beta_n)/\sum_n \theta_n \beta_n$ to get the first expression for total labor allocated to non-routine tasks:

$$L_i^a = c(1-\alpha_i)^{-\frac{1}{\mu_i}} \left(\frac{M_i}{A_i}\right)^{\mu_i} \left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} \tag{57}$$

Labor market clearing together with the production technology for the routine input (6) provides the second expression for total labor allocated to non-routine tasks:

$$L_i^a = \bar{L} - L_i^m = \bar{L} - (1-\alpha_i)^{-\frac{1}{\mu_i}} \left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1}{\mu_i}} \tag{58}$$

Combining these two expressions gives:

$$\bar{L} - (1-\alpha_i)^{-\frac{1}{\mu_i}} \left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1}{\mu_i}} = c(1-\alpha_i)^{-\frac{1}{\mu_i}} \left(\frac{M_i}{A_i}\right)^{\mu_i} \left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} \tag{59}$$

Rearranging to factor out $(1-\alpha_i)^{-\frac{1}{\mu_i}} \left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1}{\mu_i}}$ gives:

$$\bar{L} = (1-\alpha_i)^{-\frac{1}{\mu_i}} \left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1}{\mu_i}} \left[1 + \frac{c\left(\frac{M_i}{A_i}\right)^{\mu_i}}{\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}}\right] \tag{60}$$

Rearranging the term in square brackets gives:

$$\bar{L} = (1-\alpha_i)^{-\frac{1}{\mu_i}} \left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} \left[(1+c)\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right] \tag{61}$$

The above expression indicates that for any given choice of endowments $\{\bar{L}, \bar{K}\}$ and parameters $\{\mu_i, c, \alpha_i, A_i\}$, there is a unique choice of routine input production that verifies FOCs for the inner and outer problems simultaneously. We have shown in 3.1.3 that routine input production M_i^* that verifies average cost minimization is compatible with any feasible choice of parameters and endowments. It follows that M_i^* must verify (61). We can confirm this assertion by plugging (15) into (61):

$$\bar{L} = (1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\bar{K}^{\mu_i}\left[\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{\frac{1-\mu_i}{\mu_i}}\left[(1+c)\left[\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}+1\right]-1\right]$$

Simplifying and rearranging gives:

$$\frac{w_i\bar{L}}{r_i\bar{K}} = (1+c)\left[1+\frac{w_i}{r_i}\left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{-\frac{1}{1-\mu_i}}\right]-1 \tag{62}$$

We plug the value of the effective labor cost (43) into (62) and find that the LHS and the RHS are indeed equal:

$$\frac{w_i\bar{L}}{r_i\bar{K}} = \frac{w_i\bar{L}}{r_i\bar{K}} - c + (1+c) - 1 \tag{63}$$

Consequently, M_i^* in (15) constitutes the unique solution to the inner and outer problems. We expect to obtain the same solution if we solve the firm problem instead of the macro problem. The idea behind the proof is the following: there is a unique solution of the relative factor price for any given set of parameters that satisfies inner and outer cost minimization. As the production function is CRS, it must be the case that the ratio $\bar{K}_i/L^{m*}$ remains unchanged if we split production among N firms rather than concentrate it within a unique firm $N(\bar{K}_i/N)/(L^{m*}/N)$. Further, it must be the case that for any given factor price ratio w_i/r_i, routine labor and capital are combined in the same way in routine input production in both sectors. Consequently, the solution of the firm problem and of the macro problem must coincide.

A.2 MC and AC curvature

The impact of differences in capital-routine labor substitutability on resource allocation can be better understood by examining the curvature and the relative position of the MC and AC functions.

The marginal cost of production is 0 when $M_i \leq A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K} = M_{min}$ since no labor is used in production up to that point. The marginal cost function is everywhere increasing for $M_i \geq M_{min}$:

$$\frac{dMC_i^m(\cdot)}{dM_i} = w_i(1-\mu_i)(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\,\frac{\bar{K}}{\mu_i}\,\frac{M_i^{\mu_i-2}}{A_i}\,\frac{M_i^{\mu_i}}{A_i}$$

$$-\alpha_i \bar{K}^{\mu_i} \qquad \frac{(1-2\mu_i)}{\mu_i} > 0$$

We denote $j = w_i(1-\mu_i)(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\left(\frac{\bar{K}}{A_i}\right)^{\mu_i}$. The second derivative of the marginal cost function is:

$$\frac{d^2MC_i^m(\cdot)}{dM_i^2} = jM_i^{\mu_i-3}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i}-\alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-3\mu_i}{\mu_i}}\left\{(2-\mu_i)\alpha_i\bar{K}^{\mu_i}-(1+\mu_i)\left(\frac{M_i}{A_i}\right)^{\mu_i}\right\}$$

The sign of the second derivative is determined by the sign of the expression in the curly brackets. The marginal cost function is convex as long as:

$$(2-\mu_i)\alpha_i\bar{K}^{\mu_i} \geq (1+\mu_i)\left(\frac{M_i}{A_i}\right)^{\mu_i} \Leftrightarrow M_i \leq \left(\frac{2-\mu_i}{1+\mu_i}\right)^{\frac{1}{\mu_i}} A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K} \tag{64}$$

The marginal cost function is convex at the intersection with the AC curve whenever:

$$M_i^* = A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}\left[1+\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{1/\mu_i} \leq \left(\frac{2-\mu_i}{1+\mu_i}\right)^{\frac{1}{\mu_i}} A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$$

$$1+\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} \leq \frac{2-\mu_i}{1+\mu_i} \tag{65}$$

There are three possible cases. If $\mu_i \geq .5$, the expression on the RHS is smaller than one, whereby the marginal cost curve always becomes concave before or at the intersection with the average cost curve. Second, for any μ, the marginal cost curve becomes concave before the intersection with the average cost curve if the relative wage is sufficiently high ($(w_i/r_i) >$ $^{1/(1-\mu_i)}$). If none of the above holds, then it could be the case that the marginal cost function is still convex at the intersection with the AC curve.

We also characterize the curvature of the AC function:

$$\frac{d^2AC()}{d(M_i)^2} = \frac{d}{d(M_i)}\left\{\frac{w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\bar{K}^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i}-\alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}}-r_i\bar{K}}{M_i}\right\}$$

$$= \frac{w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\bar{K}^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i}-\alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-2\mu_i}{\mu_i}}\left\{2\alpha_i\bar{K}^{\mu_i}-(1+\mu_i)\left(\frac{M_i}{A_i}\right)^{\mu_i}\right\}+2r_i\bar{K}}{M_i^3} \tag{66}$$

A sufficient condition for the convexity of the AC function is:

$$2\alpha_i\bar{K}^{\mu_i} \geq (1+\mu_i)\left(\frac{M_i}{A_i}\right)^{\mu_i} \Leftrightarrow M_i \leq \left(\frac{2}{1+\mu_i}\right)^{\frac{1}{\mu_i}} A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$$

It is immediate that the inflexion point of the AC function always lies to the right of the inflexion point for the MC function. Further, it can be shown that the numerator of (66) is always positive at M_i^* whereby it follows that the inflexion point of the AC curve is situated in the increasing portion of the AC function.

A.3 Relative position of AC and MC curves

The marginal cost function is equal to the average cost function when $M_i = M_i^*$. Given $MC_i^m(M_{min}) < AC_i^m(M_{min})$ together with $dMC_i^m(\cdot)/dM_i > 0$ and $dAC_i^m(\cdot)/dM_i < 0$ for $M_i \in [M_{min}, M_i^*]$, the marginal cost function is below the average cost function for $M_i \in [M_{min}, M_i^*]$.

It remains to be shown that the marginal cost function is everywhere above the average cost function for $M_i > M_i^*$. Both functions are increasing in this range.

$$MC(M_i) > AC(M_i) \Leftrightarrow$$

$$\frac{w_i}{(1-\alpha_i)^{\frac{1}{\mu_i}}} M_i \frac{\mu_i}{A_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} > \frac{w_i}{(1-\alpha_i)^{\frac{1}{\mu_i}}} M_i \frac{\mu_i}{A_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1}{\mu_i}} + r_i\bar{K}$$

We rearrange this expression, factor out common terms and simplify to get:

$$w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\bar{K}^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} > r_i\bar{K}$$

$$\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i} > \left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\alpha_i^{-\frac{\mu_i}{1-\mu_i}}(1-\alpha_i)^{\frac{1}{1-\mu_i}}\bar{K}^{\mu_i}$$

$$M_i > A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}\left[1+\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{\frac{1}{\mu_i}} = M_i^*$$

The latter expression indeed holds. We conclude that the marginal cost function is everywhere above the average cost function beyond the point of average cost minimization.

Further, we can show that the marginal cost function constitutes an asymptote of the average cost function when $M_i \to \infty$. Consider the ratio:

$$\frac{MC(M_i)}{AC(M_i)} = \frac{w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\left(\frac{M_i}{A_i}\right)\mu_i\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}}}{w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1}{\mu_i}} + r_i\bar{K}} \tag{67}$$

We evaluate this ratio at the point in which routine input production tends to its maximum:

$$\lim_{L_i^m \to \bar{L}} M_i(L_i^m) = A_i\left[(1-\alpha_i)\bar{L}^{\mu_i} + \alpha_i\bar{K}^{\mu_i}\right]^{1/\mu_i}$$

$$\frac{MC(M_i(\bar{L}))}{AC(M_i(\bar{L}))} = \frac{w_i(1-\alpha_i)^{-1}\bar{L}^{1-\mu_i}\left[(1-\alpha_i)\bar{L}^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]}{w_i\bar{L} + r_i\bar{K}}$$

Consider the position of the two curves when $\bar{L} \to \infty$:

$$\lim MC(M_i(\bar{L})) = \lim \frac{w_i(1-\alpha_i)^{-1}\bar{L}^{1-\mu_i}\left[(1-\alpha_i)\bar{L}^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]}{} \tag{68}$$

$$\bar{L}\to\infty \quad \overline{AC\,(M_i(\bar{L}))} \qquad \bar{L}\to\infty \qquad\qquad w_i\bar{L}+r_i\bar{K}$$

Using L'Hopital's rule, this limit can be evaluated by taking the derivative of the numerator and of the denominator wrt $\bar{L}$:

$$\lim_{\bar{L}\to\infty} \frac{MC\left(M_i(\bar{L})\right)}{AC\left(M_i(\bar{L})\right)} = \lim_{\bar{L}\to\infty} \left\{ \mu_i + (1-\mu_i)\left[1 + \frac{\alpha_i}{1-\alpha_i}\left(\frac{\bar{K}}{\bar{L}}\right)^{\mu_i}\right]^{-1} \right\} = 1 \tag{69}$$

The second term in the square brackets approaches 0 from above. Consequently, the AC function converges to the MC function from below as $M_i \to \infty$.

B Alternative approach to solving the benchmark model

B.1 The problem of the firm in routine input production

The cost minimization problem of the firm is:

$$\begin{cases} \text{Min } w_i L_i^m + r_i K_i \\ s.t.\ \ M_i \leq A_i \left[\alpha_i (K_i)^{\mu_i} + (1-\alpha_i)(L_i^m)^{\mu_i}\right]^{1/\mu_i} \end{cases}$$

The first order conditions define relative factor demand as a function of the factor price ratio:

$$\frac{L_i^m}{K_i} = \left[\frac{w_i}{r_i}\frac{\alpha_i}{1-\alpha_i}\right]^{-\frac{1}{1-\mu_i}} \tag{70}$$

We rearrange this expression to solve for each of the factors and plug it into the production function to obtain conditional factor demands:

$$K_i = \frac{M_i}{A_i}\left[\alpha_i + (1-\alpha_i)\left(\frac{w_i\,\alpha_i}{r_i\,(1-\alpha_i)}\right)^{\frac{\mu_i}{\mu_i-1}}\right]^{-\frac{1}{\mu_i}} = \frac{M_i}{A_i}[\alpha_i]^{-\frac{1}{\mu_i}}\left[1 + \left(\frac{w_i}{r_i}\right)^{\frac{\mu_i}{\mu_i-1}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}$$

$$L_i^m = \frac{M_i}{A_i}\left[(1-\alpha_i) + \alpha_i\left(\frac{w_i\,(1-\alpha_i)}{r_i\,\alpha_i}\right)^{\frac{\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}} = \frac{M_i}{A_i}[1-\alpha_i]^{-\frac{1}{\mu_i}}\left[1 + \left(\frac{w_i}{r_i}\right)^{\frac{\mu_i}{1-\mu_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}$$

We rearrange each of the expressions in square brackets. In the capital equation we factor out $\alpha_i^{-\frac{1}{1-\mu_i}} r_i^{\frac{\mu_i}{1-\mu_i}}$. In the routine labor equation we factor out $(1-\alpha_i)^{-\frac{1}{1-\mu_i}} w_i^{\frac{\mu_i}{1-\mu_i}}$.

For capital, we get:

$$\begin{aligned} K_i(M_i; w_i, r_i) &= \frac{M_i}{A_i}[\alpha_i]^{-\frac{1}{\mu_i}}\left[\alpha_i^{-\frac{1}{1-\mu_i}} r_i^{\frac{\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}} \\ &= \frac{M_i}{A_i}\left[\frac{\alpha_i}{r_i}\right]^{\frac{1}{1-\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}} \end{aligned}$$

For routine labor, we get:

$$\begin{aligned} L_i^m(M_i; w_i, r_i) &= \frac{M_i}{A_i}[1-\alpha_i]^{-\frac{1}{\mu_i}}\left[(1-\alpha_i)^{-\frac{1}{1-\mu_i}} w_i^{\frac{\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}} \\ &= \frac{M_i}{A_i}\left[\frac{1-\alpha_i}{w_i}\right]^{\frac{1}{1-\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1 \end{aligned}$$

$$- \alpha_i) w_i \tag{71}$$

We plug the conditional factor demands in the cost of production to obtain the unit cost function:

$$C(w_i, r_i; 1) = \frac{1}{A_i}\left[\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}} \left[(1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{-\frac{\mu_i}{1-\mu_i}} + \alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{1-\frac{1}{1-\mu_i}}$$

$$C(w_i, r_i; 1) = \frac{1}{A_i}\left[\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{\frac{\mu_i-1}{\mu_i}} \tag{72}$$

The price of the routine input coincides with the solution of the macro problem (19).

B.2 Autarky Equilibrium

The resource constraint on capital puts an upper bound on the maximal amount of the routine input that can be produced by optimally combining capital and labor:

$$K_i^* = \frac{M_i}{A_i}\left(\frac{\alpha_i A_i P_i^m}{r_i}\right)^{\frac{1}{1-\mu_i}} \leq \bar{K} \Leftrightarrow M_i \leq \bar{K} A_i \left(\frac{\alpha_i A_i P_i^m}{r_i}\right)^{-\frac{1}{1-\mu_i}} \tag{73}$$

The level of production of the routine input coincides with the solution of the macro problem whenever the capital endowment is fully used ((74) replicates (15)):

$$M_i^* = \bar{K} A_i \left(\frac{\alpha_i}{r_i}\right)^{-\frac{1}{1-\mu_i}} \left[\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{\frac{1}{\mu_i}}$$

$$M_i^* = \bar{K} A_i \alpha_i^{\frac{\mu_i}{\mu_i(1-\mu_i)}} r_i^{\frac{1}{1-\mu_i}} \left[\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{\frac{1}{\mu_i}}$$

$$M_i^* = \bar{K} A_i \alpha_i^{\frac{1}{\mu_i}} \left[1 + \left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{\frac{1}{\mu_i}} \tag{74}$$

The above expression uses capital market clearing to define the optimal quantity of the routine output. We obtain the second expression for the optimal quantity of the routine output using labor market clearing. Cost minimization in the production of final goods delivers (28). We use labor market clearing to write $L_i^{a*} = \bar{L} - L_i^{m*}(M_i^*)$. We use conditional labor demand in routine output production (71) to replace L_i^{m*} by its value whereby (28) becomes:

$$\frac{\bar{L} - \frac{M_i}{A_i}\left(\frac{1-\alpha_i}{w_i}\right)^{\frac{1}{1-\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}}{M_i^*} = c\frac{P_i^m}{w_i} \tag{75}$$

We factor out M_i^* on the LHS, replace P_i^m by its value in (72) on the RHS, rearrange the expression and solve for M_i^* to get:

$$\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{-\frac{\mu_i}{1-\mu_i}}\Big]^{\frac{1}{\mu_i}}$$

$$M_i^* = \frac{A_i w_i \bar{L}}{c\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1+c)(1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{\frac{-\mu_i}{1-\mu_i}}} \tag{76}$$

We equate (76) with the first line of (74) to get:

$$\bar{K}\left(\frac{\alpha_i}{r_i}\right)^{\frac{1}{1-\mu_i}} = w_i\bar{L}\left[c\alpha_i^{\frac{1}{1-\mu_i}} r_i^{-\frac{\mu_i}{1-\mu_i}} + (1+c)(1-\alpha_i)^{\frac{1}{1-\mu_i}} w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{-1}$$

We factor out $\alpha_i^{1/(1-\mu_i)} r_i^{-\mu_i/(1-\mu_i)}$ and simplify the above expression to obtain an implicit solution for the factor price ratio $\omega_i^* = (w_i/r_i)^*$:

$$(\omega_i^*)^{-1} c + (1+c)\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} (\omega_i^*)^{-\frac{1}{1-\mu_i}} - \frac{\bar{L}}{\bar{K}} = 0 \qquad (77)$$

The solution to the autarky equilibrium is unchanged: (77) replicates (31).

B.3 Existence and uniqueness of the solution

As we show in 3.1.3, the polynomial in (77) has a unique positive root ω_i^* whenever the relative price of capital is not 'too high' $(r_i/w_i)^* \leq c^{-1}(\bar{L}/\bar{K})$. To investigate whether this inequality always holds, we start from some initial endowments for which it is satisfied and characterize the magnitude of the change in the factor price ratio and in the relative endowment following a positive shock to $\bar{L}/\bar{K}$.[32] Differentiating both sides with respect to $\bar{L}/\bar{K}$, we get:

$$\frac{\partial\left(\frac{r_i}{w_i}\right)^*}{\partial\left(\frac{\bar{L}}{\bar{K}}\right)} d\frac{\bar{L}}{\bar{K}} = \frac{1}{1+\sigma_i\left(\frac{w_i^*\bar{L}}{r_i^*\bar{K}}-1\right)} d\frac{\bar{L}}{\bar{K}} \leq \frac{1}{c}\, d\frac{\bar{L}}{\bar{K}} \Leftrightarrow c \leq 1+\sigma_i\left(\frac{w_i^*\bar{L}}{r_i^*\bar{K}}-1\right) \qquad (78)$$

As long as the above inequality holds, the change in the factor price ratio is smaller than the change in relative factor endowments, and the initial inequality continues to hold. The magnitude of c depends on factor shares in production of final goods and on the shares of these final goods in consumption. For simplicity, we assume that $c = 1$.[33] It is immediate that the initial inequality can be rearranged as $1 \leq w_i^*\bar{L}/r_i^*\bar{K}$ whereby (78) is verified. It follows that the polynomial has a unique positive solution for any $\bar{L}^I/\bar{K}^I \geq \bar{L}/\bar{K}$, i.e. both labor and capital continue to be used in routine input production as labor becomes more and more abundant.

The intuition is the following. An increase in the labor endowment translates into an increase in the relative cost of capital (78). Notwithstanding this increase in the cost of capital, it remains optimal to use the full amount of capital in routine input production. Indeed, (70) indicates that by increasing the amount of capital used in production we always decrease the relative cost of capital and free up labor for non-routine tasks. By freeing up labor from routine tasks,

[32] One such initial endowment point is simply $\bar{L}/\bar{K} = 1$.

[33] $c = 1$ if the two goods carry equal weight in consumption ($\theta_1 = \theta_2 = .5$) and $\beta_1 + \beta_2 = 1$.

we always increase the total quantity of final goods that can be produced, thereby making the consumer better off.

Next, we consider the change in relative endowments and in the relative factor price ratio following a positive shock to $(\bar{K}/\bar{L})$. For the initial endowments, the inequality $(w_i/r_i)^*$ $\geq c(\bar{K}/\bar{L})$ is verified. Differentiating both sides with respect to $\bar{K}/\bar{L}$, we get:

$$\frac{\partial\left(\frac{w_i}{r_i}\right)^*}{\partial\left(\frac{\bar{K}}{\bar{L}}\right)} = \frac{\left(\frac{w_i^*\bar{L}}{r_i^*\bar{K}}\right)^2}{c + \frac{1}{1-\mu_i}(1+c)\left\{\left(\frac{1}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\left(w_i^*\right)^{-\frac{\mu_i}{1-\mu_i}}\right\}} \geq c \tag{79}$$

From the polynomial we know that the expression in the curly brackets of (79) is equal to $[w_i\bar{L}/r_i\bar{K} - c]$. Rearranging and simplifying the above expression, we get:

$$\frac{\partial\left(\frac{w_i}{r_i}\right)^*}{\partial\left(\frac{\bar{K}}{\bar{L}}\right)} = \frac{(1-\mu_i)\left(\frac{w_i^*\bar{L}}{r_i^*\bar{K}}\right)^2}{\frac{w_i^*\bar{L}}{r_i^*\bar{K}} - \mu_i c} \geq c \tag{80}$$

Again we set $c = 1$, and simplify the above expression to get:

$$\frac{(1-\mu_i)\left(\frac{w_i^*\bar{L}}{r_i^*\bar{K}}\right)^2}{\frac{w_i^*\bar{L}}{r_i^*\bar{K}} - \mu_i} \geq 1 \iff \frac{w_i^*\bar{L}}{r_i^*\bar{K}} \geq \frac{\mu_i}{1-\mu_i} \tag{81}$$

As long as the above inequality holds, the change in the factor price ratio exceeds the change in relative factor endowments, and the initial inequality always holds. The above inequality is necessarily verified if $\mu_i \leq .5$. However, the inequality may be violated for $\mu_i > .5$ whereby the initial inequality may be violated for high enough μ and sufficiently abundant capital. The intuition is straightforward. As capital endowment increases, the use of labor in routine tasks becomes more and more expensive. If μ is sufficiently high, we may reach a situation where capital becomes sufficiently cheap to fully replace labor in routine tasks.

If one or both countries stop using labor in routine input production, its price becomes $P_i^m = r_i\bar{K}/M_i$ where $M_i = A_i\alpha_i^{1/\mu_i}\bar{K}$ whereby $P_i^m = r_i/A_i\alpha_i^{1/\mu_i}$. If this approach to production is cost-minimizing, it must be that the price of the routine input is lower without using labor:

$$
\begin{aligned}
\frac{r_i}{A_i \alpha_i^{1/\mu_i}} \leq \frac{r_i}{A_i \alpha_i^{1/\mu_i}} \left[1 + \frac{w_i}{r_i} \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{-\frac{1}{1-\mu_i}} \right]^{\frac{\mu_i - 1}{\mu_i}} \Leftrightarrow \\
\left[1 + \left(\frac{w_i}{r_i} \right)^{-\frac{\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i} \right)^{\frac{1}{1-\mu_i}} \right]^{\frac{-(1-\mu_i)}{\mu_i}} \geq 1
\end{aligned}
\tag{82}
$$

The LHS of (82) is strictly smaller than 1 as long as w_i/r_i is finite. The LHS converges to 1 when $w_i/r_i \to \infty$. We conclude that when capital endowment becomes sufficiently abundant and $\mu > .5$, the weight of labor in routine input production becomes negligible. In the latter case, $L_i^a \to \bar{L}^I$, $M_i \to A_i \alpha_i^{1/\mu_i} \bar{K}^I$, and $P_i^m = r_i/A_i \alpha_i^{1/\mu_i}$ whereby (28) becomes:

$$\frac{L_i^{a*}}{M_i} = c\frac{P_i^m}{w_i} \Leftrightarrow \frac{\bar{L}^I}{\bar{K}^I} = c\frac{r_i}{w_i} \Leftrightarrow \omega_i^* = c\frac{\bar{K}^I}{\bar{L}^I} \tag{83}$$

This situation must occur in the high-μ country before the low-μ country because the equilibrium factor price ratio $\omega^*(\mu_1) < \omega^*(\mu_2)$ when capital endowment increases relatively to the point of normalization. It follows that $\frac{\bar{K}^I}{\bar{L}^I}$ for which $\omega^*(\mu_1) \to c_1\frac{\bar{K}^I}{\bar{L}^I}$ has $\omega^*(\mu_2) > c_2\frac{\bar{K}^I}{\bar{L}^I}$. As the relative wage is lower in the high-μ country, this country continues to have a relatively lower autarky price for the non-routine intensive final good.

If $\mu_2 > .5$ and the capital endowment continues to increase, the low-μ country also reaches the point where only capital is used in routine input production. Beyond this threshold, differences in capital-labor substitutability cease to be a source of comparative advantage.

To sum up, we have a unique positive solution to the polynomial for any factor endowments if $\mu_i \leq .5$, and the pattern of specialization described in the core of the paper always holds. Whenever both μ_1 and μ_2 are strictly bigger than .5, there exists a threshold at which the relative capital endowment is sufficiently high for labor to become negligible in routine input production. In the latter case, our mechanism ceases to be a source of comparative advantage.